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Determination and analysis of the orbit of Meteor 28 (1977-57A) at 154 epochs during 15th-order resonance

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The orbit of Meteor 28 (1977-57A) has been determined at 154 epochs between 8 January 1984 and 18 July 1989 during which the orbit was perturbed by 15th-order resonance with the geopotential. In total 9521 observations were used in the orbit determination process, the majority of which were supplied by the U.S. Navy. The remainder were British radar, visual and Hewitt camera observations. The variations in the inclination and eccentricity have been analysed to determine six lumped

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harmonics of order 15 and four of order 30 with accuracies of between 0.4 and 4.4 cm in geoid height. Since the orbit of Meteor 28 was Sun-synchronous, many sources of perturbation of the orbit (solar gravity, tidal effects, solar radiation pressure and drag) were enhanced by solar resonance. The removal of perturbations from the inclination and eccentricity to isolate the geopotential resonance effects posed a significant challenge in the analysis.

1. Introduction

Meteor 28 (1977-57A) was launched on 29 June 1977 and was the first Soviet satellite to be placed into a retrograde orbit. Its initial inclination was 97.91° with a perigee height of 601 km, apogee height 670 km and nodal period of 97.46 min (King-Hele *et al.* 1991). This choice of orbit was such that the nodal regression rate was identical to the motion of the Earth in its orbit around the Sun (0.986 deg day^{-1}). Such an orbit is said to be Sun-synchronous and has the property of passing over a given point of the Earth's surface at the same local solar time throughout the year. The purpose of the Meteor satellites was primarily for weather monitoring, Meteor 28 being an experimental version. The spacecraft consisted of a cylindrical body with two planar solar arrays erected one on either side. It passed through exact 15th-order resonance with the geopotential on 11 July 1988 under the influence of, initially low, air drag.

The purpose of this work is to determine lumped harmonics of the Earth's geopotential by analysing the changes in inclination and eccentricity of the orbit as it passed through resonance. U.S. Navy radar observations supplied by the U.S. Naval Research Laboratory, British radar, together with Hewitt camera and visual observations have been utilized to determine the orbit at 154 epochs between 8 January 1984 and 18 July 1989. The MACPROP version (Swinerd 1982) of the Royal Aerospace Establishment (RAE) orbit determination program PROP 6 (Gooding 1974) was used for this purpose. Before the changes in inclination and eccentricity due to resonance with the geopotential could be analysed, other major perturbing forces had to be taken into account. The perturbations considered here were those due to the Earth's zonal harmonic and $J_{2,2}$ tesseral harmonic fields, direct luni-solar gravity, the solid-Earth and oceanic tides, solar radiation pressure and aerodynamic drag. Many of these effects would normally be considered negligible, since perturbation magnitudes are usually below the 1σ error in the measured orbit elements. However, the Sun-synchronous orbit meant that the magnitudes of these usually small perturbations were enhanced due to solar resonance; in particular the effects of the direct solar gravity, the solid-Earth and oceanic tides, solar radiation pressure and the influence of the atmosphere's diurnal density bulge in the air drag are significant.

2. The observations and their accuracies

Over 10000 observations were available for the orbit determination and this is prolific for such an analysis. Of these, 9521 were accepted, the majority of which (7879) were supplied by the U.S. Naval Research Laboratory. These were radar observations and had an average *a priori* accuracy of 2 arcmin in angular position and 1 km in range. Of the remainder, 229 were British radar observations, 390 were made by the Hewitt cameras and 1023 were visual observations obtained by the

Table 1. *Analysis of residuals for stations with more than five observations accepted*

station		number of observations accepted	range	r.m.s. residuals (arc min)		
no.	name		km	r.a.	dec.	total
1	U.S. Navy	1152		1.19	1.60	1.99
2	U.S. Navy	349		1.40	2.01	2.45
3	U.S. Navy	350		1.30	1.69	2.14
4	U.S. Navy	312		1.70	1.90	2.55
5	U.S. Navy	683		1.53	1.86	2.41
6	U.S. Navy	703		1.51	1.91	2.43
29	U.S. Navy	4330	0.6	0.20[a]	0.29[a]	0.35
341	British radar	32	0.7	2.30	2.00	3.05
342	British radar	90	1.2	2.20	2.80	3.56
343	British radar	107	1.2	2.00	2.20	2.97
414	Bergvliet	94		1.81	2.10	2.78
2115	Yatel	5		7.10	8.30	10.92
2122	Malvern 5	20		1.70	2.00	2.62
2160	Wormhout	7		7.22	4.70	8.61
2265	Farnham	90		3.37	3.36	4.76
2392	Cowbeech	6		0.41	0.99	1.07
2414	Bournemouth	254		3.94	4.29	5.83
2418	Sunningdale	24		2.37	2.83	3.69
2420	Willobrae	132		3.93	3.83	5.49
2430	Stevenage	6		1.94	1.30	2.34
2431	Copthorne	5		0.44	1.94	1.99
2437	Warrington	11		5.40	4.60	7.09
2572	Wittenheim	6		5.17	5.60	7.62
2657	Bridgwater	13		2.79	2.17	3.52
2658	Hillsborough	20		3.11	3.59	4.75
2659	Herstmonceux Hewitt Camera	106		0.27	0.27	0.38
2662	Northwood	94		3.11	4.05	5.11
2665	Ciresilor	129		3.57	4.63	5.85
2668	North Hykeham	5		4.89	3.64	6.09
2669	Wien, Austria	5		1.34	1.89	2.32
2675	Malvern Link	28		2.33	2.12	3.14
2676	Goostrey	10		3.90	2.33	4.54
2691	RAF Ascension	20		2.06	3.84	4.35
4160	Achel, Belgium	16		4.25	6.96	8.15
8517	Davis, U.S.A.	10		1.43	2.05	2.50
8544	North Canton	9		1.90	2.00	2.76
9652	Siding Spring Hewitt Camera	284		0.09	0.08	0.12

[a] Geocentric.

network of volunteer observers who currently report to the Royal Greenwich Observatory. The average *a priori* accuracy of the British radar observations was 3 arcmin in angular position and 1 km in range. The visual observations had an expected accuracy of between 3 and 5 arcmin. Two sources of Hewitt camera observations were available, one being located at the Royal Greenwich Observatory, Herstmonceux, U.K. The other was at Siding Spring, Australia and this was additionally significant in that it was a valuable source of observations from the

Southern Hemisphere. The Hewitt cameras were by far the most accurate sources of observation, having an expected accuracy of between 0.01 and 0.06 arcmin in right ascension and declination and 0.1 ms in time.

The residuals of the observations have been computed and tabulated by using the MACORES program (Swinerd 1982) and distributed to the observers. The results are summarized in table 1. The angular residuals for station 29 are geocentric and need to be multiplied by a factor of approximately 5 to compare them to the topocentric values. The contribution of the volunteer observers network is acknowledged and very much appreciated.

3. The orbits and their accuracies

The orbits have been determined at 154 epochs between 8 January 1984 and 18 July 1989 using the MACPROP version of the RAE orbit determination program PROP6. The orbital elements for each epoch are listed in table 2, together with their standard deviations below each value, the epoch being at 00 h on the day indicated. The mean anomaly, M, is fitted by a polynomial of the form

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + M_4 t^4 + M_5 t^5, \tag{1}$$

where t is the time from each epoch. The number of M coefficients required depends on the atmospheric drag during the time span of the orbit determination and in most cases only M_0–M_2 were required. Some epochs did, however, need the addition of the M_3 term to produce a satisfactory fit when the drag was more significant.

The parameter ϵ indicates the measure of fit of the observations to the orbits and this varied between 0.19 and 0.98. The standard deviations in inclination varied between 0.00004° and 0.00181°, equivalent to an uncertainty of around 5 m and 220 m respectively in orbital position. For the eccentricity, the standard deviations varied between less than 0.000001 and 0.000018 corresponding to about 7 m and 125 m respectively in perigee distance. The average accuracy of the M_1 terms of equation (1) was 0.00020 deg d^{-1}. The accuracy of the M_2 terms varied generally between 0.1 % and 10.0 %. However some 6 % of the orbits have an accuracy for M_2 marginally above the 10 % level. The worst case is epoch 94 (MJD 46975) where the level of error in M_2 rises to 28 %. This epoch posed some difficulty at the orbit determination stage; the addition of an M_3 term produced a worse result with a greater error in M_2 and an indeterminate M_3 coefficient. After some effort, the presented orbit was thought to be the best possible. The M_3 coefficients, where used, have an accuracy of 10 % or better.

4. Resonance theory

(a) *Introduction*

The Earth's gravitational potential may be expressed as a double infinite series of tesseral harmonics of degree l and order m, with $m \leqslant l$. The order of the harmonic specifies the variation of the potential with longitude and in simple terms a satellite orbit will experience mth-order resonance if its ground-track repeats after m revolutions. The orbit will be significantly perturbed by the mth-order harmonics in the Earth's geopotential if its rate of decay under the influence of air drag is slow enough. Meteor 28 passed through 15th-order resonance very slowly, but the pace accelerated when the air drag increased towards the end of the analysis period. The

Table 2. *Orbital parameters of Meteor 28 (1977-57A) at the 154 epochs, with standard deviations*

(MJD, modified Julian day; a, semi-major axis (kilometres); e, eccentricity; i, inclination (degrees); Ω, right ascension of the ascending node (degrees); ω, argument of perigee (degrees); M_0, mean anomaly (degrees); M_1, mean motion (degrees day^{-1}); M_2, M_3, other coefficients required to fit polynominal for M; ϵ, measure of fit; N, number of observations used; D, time covered by observations (days).)

	MJD date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ϵ	N	D
1	45707	6950.3565	0.004546	97.60150	247.9022	77.7647	319.2688	5395.55624	0.002452	—	0.52	55	10
	8 Jan. 84	0.0002	0.000007	0.00108	0.0008	0.0976	0.0976	0.00022	0.000087				
2	45736	6950.1379	0.003210	97.59921	276.1770	357.3500	176.8094	5395.81099	0.002087	—	0.48	52	10
	6 Feb. 84	0.0002	0.000008	0.00113	0.0008	0.1207	0.1209	0.00022	0.000078				
3	45775	6949.6796	0.003038	97.59691	314.1828	181.9382	65.9188	5396.34505	0.008316	—	0.25	60	10
	16 Mar. 84	0.0001	0.000006	0.00056	0.0004	0.0601	0.0599	0.00010	0.000036				
4	45816	6949.1877	0.004626	97.58662	354.1089	73.2973	258.5153	5396.91851	0.009496	—	0.79	99	10
	26 Apr. 84	0.0003	0.000011	0.00128	0.0014	0.1608	0.1592	0.00031	0.000120				
5	45836	6948.9717	0.003645	97.58345	13.5777	19.5215	186.3372	5397.17034	0.003042	—	0.65	70	10
	15 May 84	0.0002	0.000015	0.00110	0.0009	0.1423	0.1417	0.00027	0.000098				
6	45867	6948.7796	0.002264	97.58082	43.7343	244.6276	132.4605	5397.39433	0.004619	—	0.62	72	10
	16 Jun. 84	0.0002	0.000010	0.00123	0.0010	0.3637	0.3646	0.00027	0.000098				
7	45889	6948.5881	0.003539	97.58057	65.1352	158.3504	89.8050	5397.61760	0.002443	—	0.59	100	10
	8 Jul. 84	0.0002	0.000012	0.00083	0.0009	0.1323	0.1315	0.00021	0.000073				
8	45920	6948.5397	0.004456	97.57044	95.2753	80.9379	349.6478	5397.67430	0.001979	—	0.57	99	10
	8 Aug. 84	0.0002	0.000005	0.00075	0.0006	0.1486	0.1498	0.00021	0.000077				
9	45949	6948.3592	0.003211	97.57046	123.4565	1.7952	266.2050	5397.88459	0.003797	—	0.51	80	10
	6 Sep. 84	0.0002	0.000011	0.00094	0.0007	0.0914	0.0916	0.00020	0.000069				
10	46000	6948.2198	0.004010	97.55692	172.9678	146.6845	203.3318	5398.04749	0.004831	—	0.43	79	10
	27 Oct. 84	0.0002	0.000007	0.00072	0.0005	0.0858	0.0853	0.00018	0.000059				
11	46027	6948.1171	0.004528	97.55451	199.1679	77.1974	131.4582	5398.16727	−0.000574	—	0.50	74	10
	23 Nov. 84	0.0002	0.000005	0.00086	0.0006	0.1010	0.1005	0.00019	0.000056				
12	46045	6948.0888	0.003707	97.55158	216.6216	30.6285	84.4132	5398.20042	0.001807	—	0.28	57	10
	11 Dec. 84	0.0001	0.000005	0.00057	0.0005	0.0433	0.0434	0.00012	0.000048				
13	46089	6947.8960	0.002442	97.55188	259.2941	208.1474	44.8102	5398.42512	−0.000840	—	0.52	75	10
	24 Jan. 85	0.0002	0.000008	0.00094	0.0006	0.1152	0.1154	0.00021	0.000062				
14	46112	6947.9143	0.004064	97.54525	281.5856	136.4490	2.2074	5398.40395	0.000452	—	0.35	76	10
	16 Feb. 85	0.0001	0.000005	0.00057	0.0004	0.0701	0.0700	0.00010	0.000038				
15	46133	6947.8426	0.004705	97.54385	301.9277	82.7634	312.1020	5398.48766	0.003672	—	0.56	59	10
	9 Mar. 85	0.0002	0.000007	0.00115	0.0008	0.1637	0.1629	0.00027	0.000089				
16	46178	6947.7635	0.002405	97.53157	345.4976	291.2558	247.4097	5398.58017	0.001507	—	0.45	100	10
	23 Apr. 85	0.0001	0.000005	0.00066	0.0006	0.1934	0.1940	0.00014	0.000055				

Table 2. (*cont.*)

	MJD date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ϵ	N	D
17	46196	6947.7126	0.002840	97.53072	2.9037	202.5988	250.1972	5398.63954	0.002446	—	0.85	82	10
	11 May 85	0.0002	0.000011	0.00078	0.0015	0.1367	0.1360	0.00026	0.000093				
18	46209	6947.6301	0.003734	97.53113	15.4764	157.4086	234.3270	5398.73574	0.004364	—	0.42	86	10
	24 May 85	0.0001	0.000008	0.00066	0.0005	0.0779	0.0775	0.00016	0.000046				
19	46222	6947.5570	0.004392	97.52945	28.0508	122.0658	209.9777	5398.82101	0.001307	—	0.82	99	10
	6 Jun. 85	0.0002	0.000011	0.00119	0.0011	0.2049	0.2038	0.00023	0.000079				
20	46232	6947.5518	0.004552	97.53108	37.7213	96.6449	189.9225	5398.82703	−0.000999	—	0.55	86	10
	16 Jun. 85	0.0002	0.000005	0.00072	0.0006	0.1669	0.1661	0.00018	0.000068				
21	46242	6947.5655	0.004393	97.52568	47.3843	72.8464	168.0900	5398.81125	−0.000863	−0.0003317	0.60	63	10
	26 Jun. 85	0.0004	0.000008	0.00115	0.0008	0.2114	0.2105	0.00045	0.000117	0.0000332			
22	46252	6947.5759	0.003947	97.52360	57.0505	47.0313	148.0262	5398.79913	0.001011	—	0.38	83	10
	6 Jul. 85	0.0001	0.000006	0.00063	0.0005	0.1017	0.1012	0.00014	0.000049				
23	46262	4947.5526	0.003362	97.52285	66.7108	20.4547	128.9342	5398.82627	0.001522	—	0.45	94	10
	16 Jul. 85	0.0001	0.000009	0.00068	0.0005	0.1037	0.1030	0.00014	0.000049				
24	46272	6947.5263	0.002725	97.52319	76.3702	349.7453	114.2574	5398.85691	0.001885	—	0.52	94	10
	26 Jul. 85	0.0002	0.000013	0.00105	0.0009	0.1552	0.1558	0.00025	0.000082				
25	46282	6947.4701	0.002014	97.52199	86.0316	308.7811	110.2940	5398.92248	0.003729	—	0.54	100	10
	5 Aug. 85	0.0001	0.000007	0.00079	0.0005	0.2622	0.2628	0.00015	0.000061				
26	46292	6947.4088	0.001853	97.52228	95.6942	255.0410	119.8396	5398.99405	0.003028	—	0.45	100	10
	15 Aug. 85	0.0001	0.000004	0.00053	0.0005	0.2625	0.2630	0.00013	0.000047				
27	46302	6947.3907	0.002393	97.52128	105.3566	206.6924	124.5233	5399.01512	−0.000491	—	0.80	77	10
	25 Aug. 85	0.0002	0.000018	0.00110	0.0010	0.2590	0.2597	0.00021	0.000080				
28	46312	6947.4144	0.003135	97.51624	115.0134	172.8246	114.6377	5398.98764	−0.001983	—	0.59	63	10
	4 Sep. 85	0.0002	0.000016	0.00092	0.0007	0.1211	0.1206	0.00023	0.000087				
29	46322	6947.4359	0.003747	97.51365	124.6684	144.5448	98.8292	5398.96262	−0.000307	—	0.52	71	10
	14 Sep. 85	0.0002	0.000011	0.00096	0.0008	0.1145	0.1141	0.00020	0.000074				
30	46332	6947.4321	0.004236	97.50978	134.3152	118.9424	80.2597	5398.96721	0.000689	—	0.36	51	10
	24 Sep. 85	0.0001	0.000006	0.00081	0.0006	0.1079	0.1074	0.00017	0.000059				
31	46342	6947.4147	0.004508	97.50938	143.9593	93.4081	61.7481	5398.98751	0.001566	—	0.47	40	10
	4 Oct. 85	0.0002	0.000007	0.00126	0.0009	0.1618	0.1610	0.00027	0.000100				
32	46352	6947.3760	0.004513	97.50558	153.6041	69.0813	42.3456	5399.03274	0.002875	—	0.39	45	10
	14 Oct. 85	0.0002	0.000007	0.00095	0.0007	0.1293	0.1287	0.00021	0.000073				
33	46362	6947.3101	0.004160	97.50757	163.2492	44.0295	24.2737	5399.10945	0.004303	—	0.44	68	10
	24 Oct. 85	0.0001	0.000008	0.00082	0.0006	0.0894	0.0889	0.00017	0.000054				

34	46372	6947.2507	0.003579	97.50681	172.8928	15.0045	10.9776	5399.17879	0.002252	—	0.36	69	10
	3 Nov. 85	0.0001	0.000007	0.00056	0.0004	0.0607	0.0604	0.00012	0.000046				
35	46382	6947.2428	0.002899	97.50373	182.5373	339.9166	4.1543	5399.18809	−0.000613	—	0.44	56	10
	13 Nov. 85	0.0001	0.000008	0.00080	0.0006	0.1132	0.1138	0.00016	0.000060				
36	46392	6947.2591	0.002414	97.49974	192.1772	294.3080	7.7483	5399.16913	−0.000874	—	0.63	66	10
	23 Nov. 85	0.0001	0.000007	0.00086	0.0007	0.3006	0.3012	0.00017	0.000082				
37	46402	6947.2611	0.002479	97.49778	201.8115	242.3706	17.5216	5399.16686	0.000385	—	0.38	58	10
	3 Dec. 85	0.0001	0.000005	0.00076	0.0005	0.1614	0.1618	0.00015	0.000062				
38	46412	6947.2490	0.003063	97.49667	211.4429	197.8438	19.9533	5399.18100	0.001459	—	0.38	76	10
	13 Dec. 85	0.0001	0.000005	0.00063	0.0004	0.0575	0.0576	0.00014	0.000050				
39	46422	6947.2200	0.003839	97.49480	221.0730	162.6756	13.2797	5399.21495	0.001704	—	0.64	93	10
	23 Dec. 85	0.0001	0.000007	0.00069	0.0005	0.0758	0.0756	0.00014	0.000073				
40	46432	6947.1761	0.004430	97.49522	230.7031	133.0592	1.4460	5399.26605	0.003229	—	0.43	61	10
	2 Jan. 86	0.0001	0.000007	0.00064	0.0005	0.0885	0.0880	0.00017	0.000058				
41	46442	6947.1078	0.004852	97.49863	240.3373	106.6260	347.0957	5399.34567	0.004228	—	0.47	90	10
	12 Jan. 86	0.0001	0.000004	0.00063	0.0005	0.0933	0.0928	0.00011	0.000042				
42	46452	6947.0503	0.004875	97.50045	249.9716	80.5871	333.1503	5399.41270	0.002198	—	0.30	50	10
	22 Jan. 86	0.0001	0.000004	0.00073	0.0005	0.0904	0.0899	0.00015	0.000055				
43	46462	6947.0431	0.004583	97.49841	259.6069	54.3052	319.8575	5399.42111	−0.001104	—	0.48	85	10
	1 Feb. 86	0.0001	0.000005	0.00075	0.0005	0.0786	0.0782	0.00016	0.000044				
44	46472	6947.0313	0.003919	97.49513	269.2362	26.0432	308.4644	5399.43492	0.001649	−0.0004533	0.63	71	10
	11 Feb. 86	0.0004	0.000009	0.00080	0.0006	0.0773	0.0774	0.00041	0.000072	0.0000245			
45	46482	6947.0382	0.003212	97.49273	278.8638	353.0529	301.9358	5399.42698	0.000232	—	0.47	66	10
	21 Feb. 86	0.0002	0.000007	0.00082	0.0006	0.0784	0.0785	0.00018	0.000061				
46	46492	6947.0218	0.002531	97.49002	288.4876	312.3468	303.1620	5399.44611	0.001376	—	0.41	100	10
	3 Mar. 86	0.0001	0.000005	0.00057	0.0004	0.1111	0.1114	0.00011	0.000057				
47	46502	6946.9865	0.002219	97.48902	298.1091	262.7908	313.5564	5399.48731	0.001986	—	0.46	79	10
	13 Mar. 86	0.0002	0.000004	0.00072	0.0005	0.1912	0.1916	0.00021	0.000058				
48	46512	6946.9517	0.002582	97.48764	307.7285	211.7967	325.7881	5399.52798	0.002553	—	0.51	80	10
	23 Mar. 86	0.0001	0.000010	0.00062	0.0005	0.1516	0.1521	0.00012	0.000057				
49	46522	6946.8887	0.003254	97.48671	317.3503	173.8005	325.5706	5399.60152	0.003994	—	0.61	75	10
	2 Apr. 86	0.0002	0.000013	0.00115	0.0010	0.1446	0.1440	0.00026	0.000093				
50	46532	6946.8151	0.003890	97.48810	326.9718	143.7020	318.2878	5399.68734	0.003840	—	0.52	93	10
	12 Apr. 86	0.0001	0.000009	0.00077	0.0007	0.1172	0.1166	0.00016	0.000059				
51	46542	6946.7713	0.004308	97.48741	336.5943	117.5916	307.7666	5399.73845	0.001085	—	0.58	82	10
	22 Apr. 86	0.0002	0.000008	0.00085	0.0006	0.1568	0.1559	0.00017	0.000063				
52	46552	6946.7771	0.004422	97.48578	346.2126	92.4286	296.5029	5399.73172	−0.001097	—	0.46	72	10
	2 May 86	0.0001	0.000005	0.00068	0.0005	0.1504	0.1496	0.00016	0.000056				
53	46562	4946.7956	0.004225	97.47837	355.8264	68.0542	284.2562	5399.71030	−0.001115	—	0.54	80	10
	12 May 86	0.0001	0.000006	0.00073	0.0006	0.1572	0.1566	0.00015	0.000064				

Table 2. (*cont.*)

	MJD date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ϵ	N	D
54	46572	6946.7984	0.003748	97.47811	5.4362	43.3505	272.1707	5399.70707	0.000718	—	0.48	86	10
	22 May 86	0.0001	0.000008	0.00082	0.0006	0.1264	0.1258	0.00017	0.000059				
55	46582	6946.7834	0.003075	97.47599	15.0406	16.4104	262.4306	5399.72454	0.000908	—	0.37	68	10
	1 Jun. 86	0.0001	0.000008	0.00061	0.0005	0.0799	0.0796	0.00013	0.000052				
56	46592	6946.7652	0.002354	97.47501	24.6439	343.8492	258.4964	5399.74583	0.001272	—	0.33	51	10
	11 Jun. 86	0.0001	0.000008	0.00064	0.0005	0.0990	0.0992	0.00016	0.000057				
57	46602	6946.7361	0.001717	97.47498	34.2479	299.1025	267.0016	5399.77973	0.002358	—	0.41	49	10
	21 Jun. 86	0.0002	0.000008	0.00084	0.0006	0.3386	0.3393	0.00019	0.000066				
58	46612	6946.6829	0.001625	97.47626	43.8544	241.3228	289.0143	5399.84185	0.003789	—	0.40	56	10
	1 Jul. 86	0.0002	0.000007	0.00084	0.0006	0.2970	0.2975	0.00018	0.000064				
59	46622	6946.6205	0.002148	97.47747	53.4617	194.3170	300.9715	5399.91455	0.003141	—	0.32	61	10
	11 Jul. 86	0.0001	0.000007	0.00057	0.0004	0.0903	0.0905	0.00012	0.000039				
60	46632	6946.5872	0.002830	97.47926	63.0704	161.4971	299.3448	5399.95341	0.000570	—	0.38	72	10
	21 Jul. 86	0.0001	0.000009	0.00072	0.0005	0.0958	0.0955	0.00016	0.000062				
61	46642	6946.6007	0.003404	97.47554	72.6775	135.1774	291.3247	5399.93769	−0.001701	—	0.43	65	10
	31 Jul. 86	0.0002	0.000009	0.00091	0.0007	0.1558	0.1551	0.00023	0.000078				
62	46652	6946.6337	0.003765	97.47186	82.2792	111.7781	280.0647	5399.89937	−0.001870	—	0.73	75	10
	10 Aug. 86	0.0002	0.000003	0.00042	0.0004	0.1071	0.1062	0.00029	0.000078				
63	46662	6946.6512	0.003863	97.46571	91.8803	89.0362	267.8171	5399.87903	−0.000235	—	0.38	72	10
	20 Aug. 86	0.0001	0.000004	0.00076	0.0005	0.1284	0.1278	0.00014	0.000057				
64	46672	6946.6456	0.003722	97.46327	101.4726	67.2539	254.5636	5399.88563	0.000473	—	0.45	77	10
	30 Aug. 86	0.0002	0.000005	0.00069	0.0007	0.1202	0.1198	0.00018	0.000053				
65	46682	6946.6321	0.003344	97.46324	111.0635	44.1271	242.7716	5399.90136	0.000830	—	0.76	100	10
	9 Sep. 86	0.0002	0.000012	0.00126	0.0011	0.2158	0.2148	0.00027	0.000107				
66	46692	6946.6050	0.002774	97.46107	120.6550	19.0189	233.1832	5399.93304	0.001607	—	0.54	58	10
	19 Sep. 86	0.0003	0.000012	0.00088	0.0011	0.1779	0.1779	0.00030	0.000086				
67	46702	6946.5572	0.002106	97.46178	130.2466	347.6343	230.2878	5399.98885	0.003371	—	0.76	72	10
	29 Sep. 86	0.0002	0.000009	0.00076	0.0010	0.2558	0.2564	0.00026	0.000112				
68	46712	6946.4898	0.001473	97.46410	139.8390	302.0546	242.2853	5400.06733	0.003751	—	0.34	55	10
	9 Oct. 86	0.0001	0.000005	0.00076	0.0005	0.2932	0.2938	0.00016	0.000059				
69	46722	6946.4326	0.001442	97.46462	149.4325	242.0511	269.4681	5400.13404	0.002450	—	0.37	63	10
	19 Oct. 86	0.0001	0.000005	0.00069	0.0005	0.3029	0.3033	0.00017	0.000050				
70	46732	6946.4065	0.002055	97.46135	159.0242	194.0869	285.0949	5400.16458	0.001354	—	0.40	51	10
	29 Oct. 86	0.0002	0.000008	0.00089	0.0006	0.1254	0.1256	0.00025	0.000077				

71	46742	6946.4041	0.002690	97.45331	168.6146	161.9205	285.0960	5400.16759	−0.001099	—	0.77	48	10
	8 Nov. 86	0.0003	0.000012	0.00181	0.0013	0.2489	0.2487	0.00033	0.000110				
72	46752	6946.4217	0.003326	97.44949	178.1953	136.6037	278.1100	5400.14719	−0.000668	—	0.52	62	10
	18 Nov. 86	0.0002	0.000010	0.00101	0.0006	0.1563	0.1557	0.00018	0.000070				
73	46762	6946.4128	0.003819	97.44785	187.7689	114.6021	267.7243	5400.15766	0.001501	—	0.48	72	10
	28 Nov. 86	0.0002	0.000003	0.00015	0.0005	0.0764	0.0763	0.00018	0.000045				
74	46772	6946.3946	0.003994	97.44668	197.3454	91.6645	258.4917	5400.17884	0.000652	—	0.60	75	10
	8 Dec. 86	0.0000	0.000004	0.00020	0.0005	0.0726	0.0726	0.00002	0.000009				
75	46782	6946.3760	0.003934	97.44727	206.9189	67.4700	250.6880	5400.20056	0.001350	—	0.49	55	10
	18 Dec. 86	0.0002	0.000006	0.00076	0.0006	0.1695	0.1687	0.00018	0.000063				
76	46792	6946.3420	0.003585	97.44647	216.4922	43.4975	242.9678	5400.24031	0.002385	—	0.46	50	10
	28 Dec. 86	0.0002	0.000007	0.00070	0.0005	0.1179	0.1175	0.00019	0.000066				
77	46802	6946.2916	0.003001	97.44796	226.0686	17.6363	237.6263	5400.29896	0.003496	—	0.65	50	10
	7 Jan. 87	0.0002	0.000015	0.00128	0.0010	0.1650	0.1650	0.00026	0.000094				
78	46812	6946.2276	0.002325	97.45150	235.6458	344.3119	240.4320	5400.37359	0.003799	—	0.45	64	10
	17 Jan. 87	0.0001	0.000009	0.00076	0.0006	0.1123	0.1126	0.00017	0.000059				
79	46822	6946.1745	0.001800	97.45270	245.2260	298.3143	256.6449	5400.43545	0.001904	—	0.33	57	10
	27 Jan. 87	0.0001	0.000005	0.00076	0.0005	0.2475	0.2480	0.00015	0.000054				
80	46832	6946.1650	0.001833	97.45067	254.8051	243.0617	282.4948	5400.44660	−0.000610	—	0.34	49	10
	6 Feb. 87	0.0002	0.000006	0.00077	0.0006	0.2614	0.2620	0.00018	0.000057				
81	46842	6946.1889	0.002481	97.44575	264.3806	197.5931	298.4489	5400.41889	−0.001977	—	0.47	67	10
	16 Feb. 87	0.0000	0.000006	0.00018	0.0003	0.1026	0.1023	0.00013	0.000046				
82	46851	6946.2136	0.003151	97.44428	272.9959	168.5190	300.6680	5400.39010	−0.001256	—	0.41	53	8
	25 Feb. 87	0.0002	0.000004	0.00026	0.0004	0.0924	0.0923	0.00025	0.000048				
83	46865	6946.2261	0.004020	97.43892	286.3898	130.9044	296.1636	5400.37565	0.000098	—	0.75	91	10
	11 Mar. 87	0.0000	0.000004	0.00018	0.0003	0.0463	0.0461	0.00002	0.000005				
84	46875	6946.2179	0.004386	97.44021	295.9528	106.0037	290.9715	5400.38526	0.000941	—	0.80	96	10
	21 Mar. 87	0.0002	0.000004	0.00023	0.0003	0.0457	0.0458	0.00018	0.000049				
85	46885	6946.1947	0.004491	97.43605	305.5130	81.6835	285.3792	5400.41237	0.001404	—	0.76	84	10
	31 Mar. 87	0.0001	0.000007	0.00018	0.0004	0.0285	0.0293	0.00007	0.000063				
86	46895	6946.1632	0.004337	97.43824	315.0710	55.9592	281.4824	5400.44906	0.002740	—	0.57	58	10
	10 Apr. 87	0.0002	0.000010	0.00115	0.0010	0.1218	0.1216	0.00027	0.000108				
87	46905	6946.1012	0.003856	97.43588	324.6308	30.4935	277.8909	5400.52140	0.004116	—	0.50	71	10
	20 Apr. 87	0.0001	0.000010	0.00079	0.0006	0.0853	0.0851	0.00015	0.000073				
88	46915	6946.0319	0.003170	97.43788	334.1919	0.3204	279.8041	5400.60229	0.004028	—	0.30	59	10
	30 Apr. 87	0.0001	0.000007	0.00068	0.0005	0.0722	0.0724	0.00014	0.000051				
89	46925	6945.9641	0.002525	97.43869	343.7521	321.9859	290.6818	5400.68129	0.003497	—	0.32	65	10
	10 May 87	0.0001	0.000006	0.00057	0.0004	0.1206	0.1210	0.00011	0.000040				
90	46935	6945.9233	0.002234	97.43785	353.3120	273.7357	312.1516	5400.72892	0.000971	—	0.35	56	10
	20 May 87	0.0002	0.000004	0.00077	0.0005	0.2234	0.2240	0.00019	0.000062				

Table 2. (*cont.*)

	MJD date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ϵ	N	D
91	46945	6945.9197	0.002558	97.43334	2.8726	224.1668	335.1789	5400.73327	−0.000769	—	0.34	42	10
	30 May 87	0.0002	0.000009	0.00103	0.0006	0.1699	0.1703	0.00018	0.000068				
92	46955	6945.9437	0.003207	97.42991	12.4281	185.6309	347.0262	5400.70541	−0.001300	—	0.40	46	10
	9 Jun. 87	0.0001	0.000011	0.00076	0.0006	0.0816	0.0819	0.00017	0.000058				
93	46965	6945.9649	0.003943	97.42577	21.9777	154.5525	351.1311	5400.68071	−0.000731	—	0.41	48	10
	19 Jun. 87	0.0002	0.000010	0.00096	0.0007	0.0906	0.0901	0.00022	0.000070				
94	46975	6945.9682	0.004535	97.42244	31.5225	127.3274	351.2352	5400.67703	0.000137	—	0.30	65	10
	29 Jun. 87	0.0001	0.000005	0.00057	0.0004	0.0682	0.0679	0.00010	0.000038				
95	46985	6945.9637	0.004888	97.42284	41.0649	101.8735	349.5798	5400.68218	0.000662	—	0.32	58	10
	9 Jul. 87	0.0001	0.000004	0.00068	0.0005	0.0854	0.0850	0.00015	0.000051				
96	46995	6945.9471	0.004978	97.42141	50.6042	77.0559	347.4100	5400.70161	0.001381	—	0.46	47	10
	19 Jul. 87	0.0002	0.000006	0.00092	0.0007	0.1294	0.1287	0.00019	0.000076				
97	47005	6945.9091	0.004750	97.42057	60.1445	51.2475	346.5176	5400.74605	0.003152	—	0.46	43	10
	29 Jul. 87	0.0002	0.000009	0.00094	0.0007	0.1268	0.1260	0.00025	0.000084				
98	47015	6945.8584	0.004206	97.42271	69.6854	24.4866	347.1405	5400.80505	0.003027	—	0.55	49	10
	8 Aug. 87	0.0003	0.000007	0.00100	0.0009	0.0848	0.0852	0.00031	0.000113				
99	47025	6945.7908	0.003503	97.42420	79.2298	352.7629	353.3752	5400.88393	0.004496	—	0.47	81	10
	18 Aug. 87	0.0001	0.000009	0.00084	0.0006	0.0784	0.0788	0.00016	0.000061				
100	47035	6945.7078	0.002895	97.42471	88.7750	313.6609	7.8822	5400.98072	0.005153	—	0.38	77	10
	28 Aug. 87	0.0001	0.000006	0.00060	0.0005	0.1242	0.1246	0.00014	0.000052				
101	47045	6945.6409	0.002654	97.42560	98.3213	266.6086	31.2836	5401.05883	0.003247	—	0.74	100	10
	7 Sep. 87	0.0000	0.000004	0.00004	0.0001	0.0629	0.0633	0.00005	0.000010				
102	47055	6945.5933	0.003088	97.42630	107.8703	219.1223	55.7735	5401.11429	0.001667	—	0.58	94	10
	17 Sep. 87	0.0001	0.000006	0.00012	0.0001	0.0970	0.0964	0.00011	0.000084				
103	47065	6945.5823	0.003748	97.42034	117.4153	183.6897	68.5475	5401.12733	0.000112	—	0.71	100	10
	27 Sep. 87	0.0000	0.000000	0.00007	0.0001	0.0405	0.0404	0.00005	0.000014				
104	47075	6945.5858	0.004428	97.41467	126.9541	152.9284	76.6651	5401.12344	−0.000244	—	0.79	72	10
	7 Oct. 87	0.0001	0.000005	0.00019	0.0003	0.0617	0.0613	0.00009	0.000031				
105	47085	6945.5678	0.004943	97.41287	136.4894	125.5724	81.3988	5401.14443	0.001883	—	0.55	68	10
	17 Oct. 87	0.0002	0.000004	0.00065	0.0006	0.0738	0.0736	0.00024	0.000077				
106	47095	6945.5317	0.005146	97.40695	146.0194	99.3655	85.3212	5401.18678	0.002807	—	0.24	70	10
	27 Oct. 87	0.0001	0.000003	0.00055	0.0004	0.0700	0.0697	0.00012	0.000043				
107	47105	6945.4749	0.005099	97.40473	155.5441	74.7402	88.2039	5401.25303	0.004001	—	0.35	65	10
	6 Nov. 87	0.0001	0.000003	0.00061	0.0004	0.0362	0.0365	0.00015	0.000058				

108	47115	6945.4092	0.004798	97.40450	165.0669	48.0037	93.9402	5401.32979	0.003110	—	0.33	50	10
	16 Nov. 87	0.0001	0.000006	0.00080	0.0005	0.0847	0.0842	0.00017	0.000062				
109	47125	6945.3424	0.004229	97.40372	174.5895	20.6819	100.9818	5401.40766	0.004819	—	0.49	49	10
	26 Nov. 87	0.0002	0.000012	0.00099	0.0007	0.0909	0.0905	0.00019	0.000086				
110	47135	6945.2699	0.003434	97.40113	184.1139	349.0115	113.2582	5401.49241	0.003650	—	0.38	36	10
	6 Dec. 87	0.0002	0.000011	0.00093	0.0007	0.1080	0.1086	0.00019	0.000074				
111	47147	6945.1788	0.002697	97.40238	195.5410	299.8504	140.1930	5401.59860	0.005468	—	0.39	50	12
	18 Dec. 87	0.0001	0.000006	0.00074	0.0006	0.2029	0.2036	0.00013	0.000037				
112	47160	6945.0636	0.002666	97.40635	207.9251	236.6267	181.0978	5401.73295	0.006945	0.0006664	0.70	44	10
	31 Dec. 87	0.0008	0.000015	0.00177	0.0012	0.3928	0.3940	0.00088	0.000189	0.0000652			
113	47170	6944.9191	0.003182	97.40835	217.4547	194.6851	207.3196	5401.90166	0.007884	—	0.29	50	10
	10 Jan. 88	0.0001	0.000007	0.00057	0.0004	0.0648	0.0649	0.00012	0.000044				
114	47180	6944.7660	0.003880	97.41238	226.9894	160.8397	227.1531	5402.08029	0.007583	—	0.35	46	10
	20 Jan. 88	0.0001	0.000009	0.00082	0.0006	0.0772	0.0768	0.00016	0.000052				
115	47190	6944.6607	0.004453	97.41390	236.5274	132.2409	243.2800	5402.20314	0.005485	—	0.29	52	10
	30 Jan. 88	0.0001	0.000006	0.00061	0.0005	0.0773	0.0769	0.00012	0.000042				
116	47200	6944.5698	0.004766	97.41449	246.0686	106.1199	258.0428	5402.30930	0.004983	—	0.25	48	10
	9 Feb. 88	0.0001	0.000003	0.00056	0.0004	0.0742	0.0738	0.00012	0.000044				
117	47213	6944.4788	0.004729	97.41494	258.4731	72.5020	278.3205	5402.41543	0.003406	—	0.67	47	16
	22 Feb. 88	0.0002	0.000009	0.00102	0.0012	0.2096	0.2084	0.00021	0.000044				
118	47226	6944.4290	0.004134	97.40960	270.8735	37.8385	300.7285	5402.47372	0.002012	—	0.35	59	10
	6 Mar. 88	0.0001	0.000008	0.00064	0.0005	0.0938	0.0933	0.00015	0.000055				
119	47236	6944.3978	0.003445	97.40881	280.4077	7.4123	322.1507	5402.51021	0.001769	—	0.42	64	10
	16 Mar. 88	0.0001	0.000005	0.00013	0.0002	0.0502	0.0509	0.00016	0.000045				
120	47246	6944.3423	0.002749	97.40474	289.9369	328.5588	352.3626	5402.57502	0.007580	—	0.98	95	10
	26 Mar. 88	0.0002	0.000004	0.00026	0.0003	0.2611	0.2626	0.00027	0.000077				
121	47256	6944.2280	0.002393	97.39607	299.4640	281.8819	31.5290	5402.70864	0.007109	—	0.64	58	10
	5 Apr. 88	0.0002	0.000008	0.00109	0.0010	0.3470	0.3478	0.00023	0.000091				
122	47266	6944.1338	0.002722	97.39701	308.9822	230.4207	76.7303	5402.81861	0.005473	—	0.96	95	10
	15 Apr. 88	0.0002	0.000003	0.00019	0.0001	0.0840	0.0835	0.00024	0.000056				
123	47276	6944.0168	0.003288	97.39031	318.4969	190.7299	111.2903	5402.95540	0.005402	−0.0004266	0.33	63	10
	25 Apr. 88	0.0003	0.000009	0.00071	0.0005	0.0707	0.0710	0.00034	0.000056	0.0000218			
124	47286	6943.9495	0.004022	97.38657	328.0078	158.6719	139.2769	5403.03415	0.006831	—	0.76	57	10
	5 May 88	0.0003	0.000013	0.00164	0.0011	0.1465	0.1460	0.00032	0.000105				
125	47296	6943.8444	0.004608	97.38592	337.5130	130.4047	164.5908	5403.15685	0.004714	—	0.46	63	10
	15 May 88	0.0001	0.000006	0.00091	0.0007	0.0989	0.0985	0.00017	0.000072				
126	47306	6943.7715	0.004951	97.38227	347.0162	104.4637	188.5784	5403.24208	0.004063	—	0.43	53	10
	25 May 88	0.0002	0.000005	0.00099	0.0007	0.1077	0.1072	0.00019	0.000059				
127	47316	6943.6788	0.004985	97.37917	356.5153	79.1370	212.8478	5403.35038	0.006213	—	0.49	68	10
	4 Jun. 88	0.0002	0.000005	0.00109	0.0007	0.1158	0.1153	0.00020	0.000059				

Table 2. (*cont.*)

	MJD date	a	e	i	Ω	ω	M_0	M_1	M_2	M_3	ϵ	N	D
128	47326	6943.5595	0.004681	97.38031	6.0124	53.3847	238.8341	5403.48969	0.006022	—	0.48	66	10
	14 Jun. 88	0.0001	0.000007	0.00096	0.0007	0.0680	0.0677	0.00016	0.000064				
129	47336	6943.4772	0.004119	97.37714	15.5108	25.9610	267.6989	5403.58588	0.005364	0.0002625	0.40	69	10
	24 Jun. 88	0.0003	0.000006	0.00081	0.0005	0.0531	0.0531	0.00039	0.000050	0.0000212			
130	47346	6943.3660	0.003353	97.37642	25.0073	354.3313	301.8826	5403.71581	0.005502	—	0.27	59	10
	4 Jul. 88	0.0001	0.000007	0.00055	0.0004	0.0519	0.0519	0.00011	0.000048				
131	47356	6943.2777	0.002619	97.37404	34.5028	313.4463	346.4959	5403.81899	0.003677	—	0.49	60	10
	14 Jul. 88	0.0001	0.000004	0.00044	0.0005	0.1191	0.1192	0.00015	0.000071				
132	47366	6943.1987	0.002398	97.37595	43.9992	264.6869	39.8793	5403.91116	0.005642	—	0.22	68	10
	24 Jul. 88	0.0001	0.000003	0.00055	0.0004	0.1051	0.1053	0.00011	0.000039				
133	47376	6943.0872	0.002754	97.37273	53.4948	215.8103	94.5055	5404.04150	0.007079	—	0.19	51	10
	3 Aug. 88	0.0001	0.000004	0.00049	0.0003	0.0785	0.0788	0.00009	0.000030				
134	47386	6942.9647	0.003470	97.37133	62.9876	177.5106	139.9295	5404.18465	0.007552	—	0.33	66	10
	13 Aug. 88	0.0001	0.000005	0.00063	0.0005	0.0653	0.0653	0.00014	0.000050				
135	47396	6942.8528	0.004168	97.37062	72.4805	146.7637	179.1840	5404.31533	0.007357	—	0.34	54	10
	23 Aug. 88	0.0001	0.000006	0.00064	0.0005	0.0670	0.0669	0.00012	0.000048				
136	47406	6942.6665	0.004683	97.36723	81.9730	119.6656	216.3931	5404.53309	0.013146	—	0.47	50	10
	2 Sep. 88	0.0002	0.000008	0.00119	0.0008	0.1373	0.1365	0.00024	0.000080				
137	47416	6942.4710	0.004893	97.36550	91.4626	94.1613	254.4325	5404.76144	0.009967	—	0.45	63	10
	12 Sep. 88	0.0001	0.000005	0.00087	0.0007	0.1247	0.1241	0.00016	0.000067				
138	47425	6942.3059	0.004780	97.36392	100.0017	71.5858	290.0743	5404.95439	0.012691	—	0.49	60	8
	21 Sep. 88	0.0002	0.000006	0.00094	0.0008	0.1485	0.1477	0.00023	0.000120				
139	47438	6941.9117	0.004175	97.35943	112.3352	36.5468	348.0983	5405.41511	0.020678	—	0.56	82	10
	4 Oct. 88	0.0002	0.000011	0.00093	0.0009	0.1348	0.1341	0.00021	0.000098				
140	47448	6941.5417	0.003479	97.35426	121.8203	5.5917	41.3571	5405.84755	0.018771	—	0.72	67	10
	14 Oct. 88	0.0003	0.000018	0.00130	0.0011	0.1552	0.1553	0.00033	0.000129				
141	47458	6941.2217	0.002837	97.35155	131.3020	328.1482	105.2084	5406.22165	0.015168	—	0.32	55	10
	24 Oct. 88	0.0001	0.000007	0.00066	0.0005	0.1244	0.1247	0.00014	0.000052				
142	47468	6940.9295	0.002444	97.34997	140.7785	278.9364	184.1639	5406.56322	0.021057	—	0.67	58	10
	3 Nov. 88	0.0003	0.000009	0.00121	0.0011	0.4007	0.4019	0.00033	0.000128				
143	47478	6940.5689	0.002678	97.34600	150.2528	230.6985	266.0600	5406.98490	0.023157	0.0002370	0.28	63	10
	13 Nov. 88	0.0003	0.000005	0.00057	0.0005	0.1257	0.1261	0.00029	0.000053	0.0000191			
144	47488	6940.1844	0.003311	97.34582	159.7264	189.7757	345.1367	5407.43440	0.020428	0.0002195	0.40	80	10
	23 Nov. 88	0.0003	0.000009	0.00067	0.0006	0.0740	0.0740	0.00037	0.000055	0.0000218			

145	47498	6939.7938	0.004036	97.34699	169.2064	157.1491	60.2513	5407.89111	0.025017	—	0.43	82	10
	3 Dec. 88	0.0002	0.000009	0.00080	0.0007	0.0954	0.0949	0.00019	0.000072				
146	47508	6939.3429	0.004622	97.34963	178.6893	128.3772	136.3982	5408.41836	0.029563	—	0.49	70	10
	13 Dec. 88	0.0002	0.000009	0.00081	0.0006	0.1405	0.1395	0.00019	0.000085				
147	47524	6938.5076	0.005069	97.35147	193.8694	85.4883	267.5450	5409.39549	0.023900	−0.0006171	0.47	65	10
	29 Dec. 88	0.0004	0.000006	0.00095	0.0008	0.1313	0.1306	0.00051	0.000072	0.0000296			
148	47553	6936.8423	0.003705	97.34794	221.3989	2.6581	190.1233	5411.34436	0.024816	—	0.45	62	10
	27 Jan. 89	0.0002	0.000011	0.00081	0.0007	0.0789	0.0788	0.00019	0.000072				
149	47584	6934.8373	0.002630	97.35054	250.8540	226.8020	246.0286	5413.69227	0.031821	—	0.50	73	10
	27 Feb. 89	0.0002	0.000007	0.00019	0.0005	0.1020	0.1024	0.00018	0.000029				
150	47612	6932.5865	0.004490	97.35037	277.4857	130.5527	303.8376	5416.33022	0.037514	—	0.49	71	10
	27 Mar. 89	0.0002	0.000009	0.00093	0.0007	0.1277	0.1271	0.00021	0.000096				
151	47640	6930.7903	0.004646	97.34833	304.1412	57.6280	47.9124	5418.43690	0.030917	—	0.45	66	10
	24 Apr. 89	0.0002	0.000007	0.00113	0.0008	0.1267	0.1260	0.00022	0.000084				
152	47668	6929.3601	0.002857	97.34560	330.8131	329.5223	219.8577	5420.11521	0.022647	—	0.40	56	10
	22 May 89	0.0002	0.000010	0.00106	0.0008	0.1572	0.1577	0.00023	0.000081				
153	47697	6927.8136	0.003105	97.34656	358.4485	193.5721	142.0403	5421.93107	0.039579	−0.0007248	0.41	63	10
	20 Jun. 89	0.0004	0.000011	0.00067	0.0007	0.1062	0.1064	0.00048	0.000075	0.0000330			
154	47725	6926.4879	0.004651	97.34692	25.1535	109.3116	46.7824	5423.48836	0.025856	−0.0002714	0.43	73	10
	18 Jul. 89	0.0003	0.000006	0.00088	0.0006	0.1335	0.1328	0.00040	0.000061	0.0000246			

rates of change of the orbital inclination and eccentricity due to resonance may be expressed in terms of lumped harmonics $\bar{C}_m^{q,k}$ and $\bar{S}_m^{q,k}$ of the geopotential. In this section the standard theory is outlined and the lumped harmonics are defined.

The expression for the longitude-dependent part of the geopotential in normalized form for any exterior point is (Kaula 1966)

$$V = \frac{\mu}{r} \sum_{l=2}^{\infty} \sum_{m=1}^{l} \left(\frac{R}{r}\right)^l P_l^m(\cos \Psi)\{\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda\} N_{lm}, \tag{2}$$

where r is the distance from the Earth's centre, Ψ is co-latitude, λ is longitude, R the mean equatorial radius of the Earth and μ its gravitational constant. $P_{lm}(\cos \Psi)$ is the associated Legendre function of degree l and order m, $\bar{C}_{lm}$ and $\bar{S}_{lm}$ are normalized tesseral harmonic coefficients, and the normalizing factor N_{lm} is given, for $m > 0$, by

$$N_{lm}^2 = 2(2l+1)(l-m)!/(l+m)!. \tag{3}$$

If the satellite executes β revolutions while the Earth rotates α times the orbit experiences $\beta:\alpha$ resonance. The corresponding perturbations in the orbital elements depend upon the resonance angle, Φ, given by

$$\Phi = \alpha(w+M) + \beta(\Omega - \theta), \tag{4}$$

where the orbit elements have their usual notation and θ is the sidereal angle. Exact resonance occurs when $\dot{\Phi} = 0$. In the case of Meteor 28, we are interested in 15th-order resonance so $\alpha = 1, \beta = 15$.

(b) *The perturbations in inclination and eccentricity due to resonance*

Lagrange's planetary equations for the inclination, i, and eccentricity, e, are

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{1}{na^2(1-e^2)^{\frac{1}{2}}}\left[\cot i \frac{\partial U}{\partial w} - \operatorname{cosec} i \frac{\partial U}{\partial \Omega}\right], \quad \frac{\mathrm{d}e}{\mathrm{d}t} = \frac{(1-e^2)^{\frac{1}{2}}}{na^2 e}\left[(1-e^2)^{\frac{1}{2}}\frac{\partial U}{\partial M} - \frac{\partial U}{\partial w}\right], \tag{5}$$

where n is the mean motion and U is the disturbing potential. The expansion of the geopotential, V, into spherical harmonics (equation (2)) may be written in terms of keplerian elements. If the partial derivatives of V are then found and substituted into equations (5), the rates of change in the inclination and eccentricity may be found. The perturbations induced by each pair of harmonic coefficients $\bar{C}_{lm}, \bar{S}_{lm}$ near resonance are given by (Allan 1973; Gooding & King-Hele 1989)

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{n(1-e^2)^{-\frac{1}{2}}}{\sin i}\left(\frac{R}{a}\right)^l \bar{F}_{lmp} G_{lpq}(k \cos i - m) \times \mathrm{Re}\{\mathrm{j}^{l-m+1}(\bar{C}_{lm} - \mathrm{j}\bar{S}_{lm}) \exp[\mathrm{j}(\gamma\Phi - qw)]\}, \tag{6}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{n(1-e^2)^{\frac{1}{2}}}{e}\left(\frac{R}{a}\right)^l \bar{F}_{lmp} G_{lpq}[(k+q)(1-e^2)^{\frac{1}{2}} - k] \times \mathrm{Re}\{\mathrm{j}^{l-m+1}(\bar{C}_{lm} - \mathrm{j}\bar{S}_{lm}) \exp[\mathrm{j}(\gamma\Phi - qw)]\},$$

where $\bar{F}_{lmp}(i)$ is Allan's normalized inclination function (Allan 1973; Gooding & King-Hele 1989), $G_{lpq}(e)$ is the eccentricity function (Kaula 1966), $\mathrm{j} = \sqrt{-1}$ and Re denotes the real part. The indices, γ, q, k and p are integers with $\gamma = 1, 2, 3, \ldots, q = 0, \pm 1, \pm 2$ and are related by $m = \gamma\beta$, $k = \gamma\alpha - q$ and $2p = l - k$. The suffix, m, of a pair of resonant harmonics $\bar{C}_{lm}, \bar{S}_{lm}$ is determined by the choice of γ, the $\gamma = 1$ term being dominant. Successive tesseral harmonic coefficients which arise for a given (γ, q) may be grouped into lumped harmonic coefficients defined by

$$\bar{C}_m^{q,k} = \sum_l Q_l^{q,k} \bar{C}_{lm}, \quad \bar{S}_m^{q,k} = \sum_l Q_l^{q,k} \bar{S}_{lm}, \tag{7}$$

where l increases in steps of 2 from its minimum value l_0. The Q factors are given by (Gooding & King-Hele 1989)

$$Q_l^{q,k} = B_l \bar{F}_{lmp} G_{lpq} (-1)^{(l-l_0)/2} / B_{l_0} \bar{F}_{l_0 m p_0} G_{l_0 p_0 q}, \tag{8}$$

where $B_l = n(1-e^2)^{-\frac{1}{2}}(R/a)^l$, $p_0 = \frac{1}{2}(l_0 - k)$. The $Q_l^{q,k}$ may be regarded as constant for a given satellite and we note from the definition that $Q_{l_0}^{q,k} = 1$.

(c) *Methodology*

The computer program THROE (Gooding 1971) is used to obtain the lumped harmonic coefficients by fitting the observed variations in inclination or eccentricity due to resonance to that predicted by equations (6). Before doing this the observed variations must be cleared of all other significant perturbations. The linear equations linking the tesseral harmonics to the lumped harmonics are determined computationally. If a similar analysis is carried out for a sufficient number of resonant satellites at different inclinations, the resulting set of linear equations may be solved to determine the individual tesseral harmonics. This final step is not, however, the object of this paper.

5. The perturbations

(a) *Introduction*

Before the variations in inclination and eccentricity can be analysed for the effects of geopotential resonance all other significant perturbations must be removed from the data. These include the direct gravitational attraction due to the Sun and Moon, the perturbation due to the Earth's zonal and $J_{2,2}$ tesseral harmonics, air drag, solid and ocean tides and solar radiation pressure. The perturbations that are induced directly or indirectly by the Sun are all resonant due to the Sun-synchronous nature of the orbit. It was the removal of these perturbations, which under non-solar resonant conditions are generally negligible, that proved the greatest challenge in this analysis. The perturbations due to atmospheric rotation and the motion of the Earth's equatorial plane (precession) were removed by the computer program THROE in the fitting process. The perturbations due to atmospheric tides and solar radiation reflected from the Earth were estimated to be negligible and hence were not considered. If all the above-mentioned perturbations in inclination and eccentricity are removed successfully then the remaining variations will be those due to the 15th-order resonance with the geopotential.

(b) *Zonal harmonic, luni-solar and tesseral harmonic perturbations*

The computer program PROD (Cook 1973) was used to calculate the perturbations in inclination and eccentricity due to luni-solar and zonal harmonic effects using one-day integration steps and restarts, where possible, every 20 days. The restarts were necessary since the integration in PROD drifts off if longer periods are used, due to the influence of other perturbations neglected in the program. The change in inclination due to the tesseral harmonic $J_{2,2}$ was computed by the program

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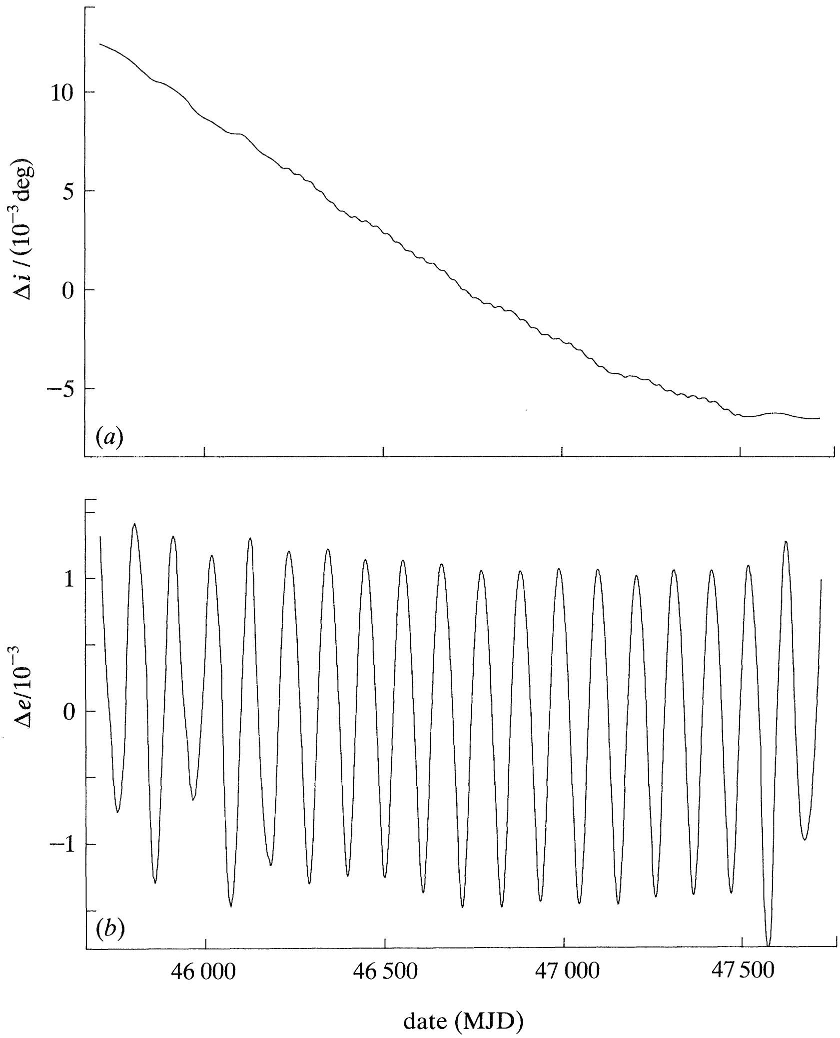

Figure 1. The combined perturbations due to zonal harmonics, the $J_{2,2}$ tesseral harmonic and direct lunisolar attraction on the inclination (*a*) and that due to zonal harmonics and direct lunisolar attraction on the eccentricity (*b*).

MACPROP as part of the orbit determination process and these values were removed from the data. The combined effects of zonal harmonics, the $J_{2,2}$ tesseral harmonic and luni-solar attraction on the inclination is shown in figure 1*a* and that due to zonal harmonics and luni-solar attraction on the eccentricity in figure 1*b*. As can be seen the perturbations in both the inclination and eccentricity are highly significant. The total change in the inclination is of the order of 0.2° over the period of the analysis and this is almost entirely caused by the near-solar resonance in solar gravitational attraction. The variation in the eccentricity is periodic with a peak-to-peak amplitude of approximately 0.003 and is predominantly due to the zonal harmonic perturbations.

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Table 3. *Amplitudes and periods of the principal solid-Earth tide perturbations at the beginning and end of the analysis and at exact 15th-order geopotential resonance*

					beginning of analysis (MJD 45707)		exact geopotential resonance (MJD 47353)		end of analysis (MJD 47725)	
l	m	p	q	tide	amp/deg	period/days	amp/deg	period/days	amp/deg	period/days
solar tides										
2	1	0	−1	S_1	-6.1×10^{-5}	−34303.9	-1.7×10^{-5}	−9924.3	-2.0×10^{-5}	−11412.5
2	1	0	0	P_1	7.7×10^{-5}	−361.4	7.3×10^{-5}	−352.3	7.4×10^{-5}	−353.9
2	1	1	0	K_1^S	7.5×10^{-5}	369.2	7.5×10^{-5}	379.2	7.5×10^{-5}	377.3
2	1	1	1	P_1	-1.8×10^{-4}	−34614.5	-4.9×10^{-5}	−9950.2	-5.7×10^{-5}	−11446.7
2	2	0	−1	T_2	1.2×10^{-5}	373.2	1.3×10^{-5}	394.2	1.2×10^{-5}	390.2
2	2	0	0	S_2	6.6×10^{-2}	−17229.3	1.9×10^{-2}	−4968.6	2.2×10^{-2}	−5714.8
2	2	0	1	R_2	8.0×10^{-5}	−357.7	7.7×10^{-5}	−340.2	7.8×10^{-5}	−343.3
2	2	1	0	K_2^S	-6.1×10^{-5}	184.6	-6.3×10^{-5}	189.6	-6.3×10^{-5}	188.7
lunar tides										
2	1	1	0	K_1^M	1.7×10^{-4}	367.3	1.9×10^{-4}	375.3	1.8×10^{-4}	373.8
2	2	0	0	M_2	1.2×10^{-4}	−14.8	1.2×10^{-4}	−14.7	1.2×10^{-4}	−14.7
2	2	0	1	N_2	1.5×10^{-5}	−9.6	1.5×10^{-5}	−9.6	1.5×10^{-5}	−9.6
2	2	1	0	K_2^M	-1.5×10^{-4}	183.6	-2.0×10^{-4}	187.7	-1.9×10^{-4}	186.9

(*c*) *Solid-Earth tides*

The most significant tidal perturbation of the satellite orbit is that due to solid-Earth tides, the solar tide being most significant due to the Sun-synchronous orbit. The potential due to solid-Earth tides may be written as (Lambeck 1977)

$$\Delta U = \sum_{d=1}^{2} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{p=0}^{l} \sum_{q=-\infty}^{\infty} \sum_{j=0}^{l} \sum_{g=-\infty}^{\infty} \Delta U_{dlmpqjg}, \tag{9}$$

where

$$\Delta U_{dlmpqjg} = k_l \left(\frac{R}{a_d}\right)^{l} \left(\frac{R}{a}\right)^{l+1} \frac{\mu_d}{a_d} (2-\delta_{0m}) \frac{(l-m)!}{(l+m)!} F_{lmp}(i_d)\, F_{lmj}(i) \times G_{lpq}(e_d)\, G_{ljg}(e) \cos(\nu_{d,\,lmpq} - \nu_{lmjg} + \epsilon_{d,\,lmpq}),$$

and

$$\nu_{d,\,lmpq} = (l-2p)\,\omega_d + (l-2p+q) M_d + m(\Omega_d - \theta),$$

$$\nu_{lmjg} = (l-2j)\,\omega + (l-2j+g) M + m(\Omega - \theta).$$

The suffix $d\,(=1,2)$ denotes the Sun and Moon respectively, δ_{0m} is the Kronecker delta, k_l are Love numbers associated with each harmonic l, and the F functions are un-normalized inclination functions (the indices l, m, p, q should not be confused with those used in §4*b*). The phase term, ϵ, may be assumed equal to zero (Lambeck 1988) and significant perturbations occur only for $l = 2$. Short-periodic terms may be eliminated by setting $l-2j+g = 0$. Furthermore, the eccentricity functions are of the order $\frac{1}{2}e^{|g|}$ and $\frac{1}{2}e^{|q|}$ and so terms beyond $g, q = \pm 1$ are generally negligible. These constraints give $j = 1, g = 0$ for the most significant terms.

Substituting the partial derivatives of equation (9) into Lagrange's planetary equations (5) and integrating gives the long-periodic perturbation in inclination due to solid-Earth tides for each d, l, m, p, q, j, g as

$$\Delta i_{dlmpqjg} = \frac{1}{na^2\sqrt{(1-e^2)}} [m \operatorname{cosec} i - (l-2j) \cot i] \frac{\Delta U_{dlmpqjg}}{\dot{\nu}_{d,\,lmpq} - \dot{\nu}_{lmjg}}. \tag{10}$$

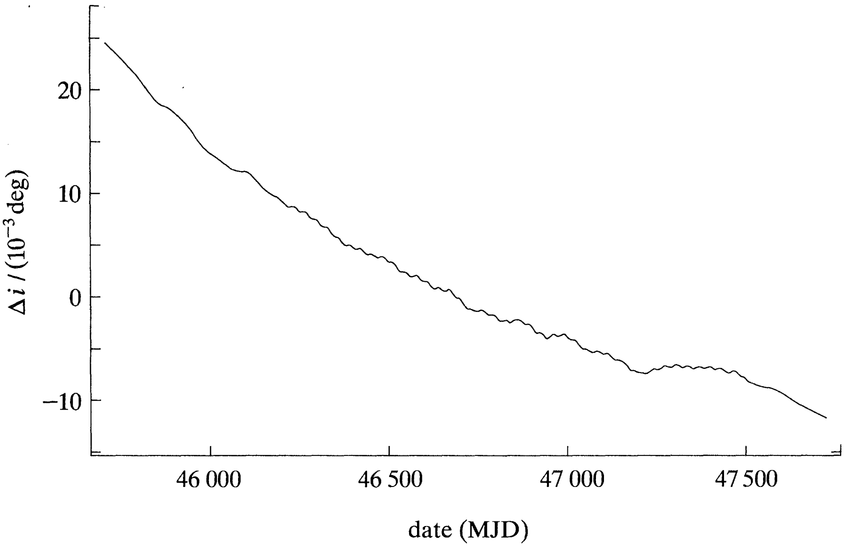

Figure 2. The perturbation in inclination due to solid-Earth tides.

Similarly, for the eccentricity

$$\Delta e_{dlmpqjg} = \frac{(1-e^2)^{\frac{1}{2}}}{na^2 e}(l-2j)\frac{\Delta U_{dlmpqjg}}{\dot{\nu}_{d,\,lmpq} - \dot{\nu}_{lmjg}}. \tag{11}$$

The restriction of significant terms to $j = 1$ and $l = 2$ ensures that the resulting long-period perturbation in e due to solid-Earth tides is negligible.

The value of k_2 used in equation (9) is 0.302. The resulting amplitudes and periods of the perturbation in inclination are shown in table 3 for the beginning and end of the analysis period and at exact 15th-order geopotential resonance. Only the most significant terms are included and it can clearly be seen that the S_2 solar tide has the largest amplitude and that it is near resonant with the orbit. The period of this perturbation is approximately 17000 days at the beginning of the analysis and approximately 6000 days towards the end. It can also be seen that the tides S_1 and P_1 are near-resonant with the orbit and have periods of approximately twice that of S_2 (*ca.* 35000 days at the beginning of the analysis). These, however, have amplitudes which are less than one-hundredth of that due to S_2. Figure 2 shows the total change in inclination due to the combined effects of the solid-Earth tides. The form of the perturbation is similar with that due to direct solar gravitation and this can be seen by comparing with figure 1*a* which is dominated by the direct solar gravitational force. This result is to be expected since both share a common origin and hence are intimately related. The amplitude of the tidal perturbation is approximately 20% of that due to the direct solar perturbation and is very significant.

(*d*) *Ocean tides*

The perturbations due to ocean tides are estimated to be up to 15% of those due to solid-Earth tides (Lambeck 1988) and so for Meteor 28 they must be taken into account. The treatment is somewhat more complicated than that due to the solid-Earth tides as the effective k_2 and phase lag ϵ (no longer negligible) are frequency dependent.

The ocean tide height for any point on the Earth with amplitude $\xi_\beta(\psi,\lambda)$ and phase $\chi_\beta(\psi,\lambda)$, both of which vary with position, may be written (Hendershott & Munk 1970)

$$\xi_\beta(\psi,\lambda,T) = \xi^0_\beta(\psi,\lambda)\cos[2\pi f_\beta T - \chi_\beta(\psi,\lambda)], \tag{12}$$

where T is mean solar time. The frequencies of the tides, f_β, are given by

$$2\pi f_\beta T = \sum_{n=1}^{6} n_i \beta_i,$$

where the β_is are the fundamental arguments of the Sun and Moon and are given as (Doodson 1921)

β_1 = local mean lunar time;

β_2 = mean longitude of the Moon;

β_3 = mean longitude of the Sun;

β_4 = longitude of lunar perigee;

β_5 = longitude of ascending node of the Moon;

β_6 = longitude of solar perigee.

There is a correspondence between the n_i of Doodson and the expansion in terms of l, m, p, q for the solid-Earth tides of equation (9) and this is discussed by Lambeck *et al.* (1973). The phase, χ_β, is with respect to the Greenwich meridian. To obtain a global representation of the ocean tide, equation (12) is expanded into spherical harmonics.

The potential, ΔU_β, outside the Earth is then given in terms of keplerian elements as (Lambeck 1977)

$$\Delta U_\beta = 4\pi G R \rho_w \sum_s \sum_t \sum_{+}^{-} \sum_u \sum_v \frac{(1+k'_s)}{2s+1}\left(\frac{R}{a}\right)^{s+1} D^{\pm}_{\beta,st} F_{stu}(i)\, G_{suv}(e)$$
$$\times \cos[2\pi f_\beta T \pm \nu_{stuv} - \epsilon^{\pm}_{\beta,st} \pm \tfrac{1}{2}(s-t)\pi], \tag{13}$$

where ν_{stuv} is as defined for the solid-Earth tides. The $D^{\pm}_{\beta,st}$ and $\epsilon^{\pm}_{\beta,st}$ are the amplitude and phase respectively for each s, t and the $+$, $-$ refer respectively to prograde and retrograde tides. The lunar and solar coordinates enter implicitly through the frequency f_β and amplitudes $D^{\pm}_{\beta,st}$. The ocean tidal perturbation in inclination due to each tidal component β and s, t, u, v is then given by

$$\Delta i_{\beta,stuv} = ((1-e^2)^{-\frac{1}{2}}/na^2)[(s-2u)\cot i - t\,\mathrm{cosec}\, i]\,4\pi \mathrm{GR}\rho_w(1+k'_s)/(2s+1)$$
$$\times (R/a)^{s+1} D^{\pm}_{\beta,st} F_{stu}\, G_{suv} \cos[2\pi f_\beta T \pm \nu_{stuv} - \epsilon^{\pm}_{\beta,st} \pm \tfrac{1}{2}(s-t)\pi]/(2\pi f_\beta \pm \dot{\nu}_{stuv}). \tag{14}$$

In a similar manner to that for the solid Earth tides, the perturbation in eccentricity due to ocean tides may be shown to be negligible.

Lambeck has shown (1977) that, unless there is resonance with a retrograde tide, long period perturbations will not occur due to the $D^{-}_{\beta,st}$. Only the prograde tides need to be considered, the most significant being diurnal ($t = 1$) and semi-diurnal ($t = 2$). The elimination of short-periodic terms requires that $s - 2u + \nu = 0$ and for small eccentricity we can further assume that terms beyond $\nu = 0$ are negligible. These constraints require s to be even, the most significant terms being $s = 2, 4, 6$. The oceanic parameters $D^{+}_{\beta,st}$, $\epsilon^{+}_{\beta,st}$ are obtained from oceanic models or from other

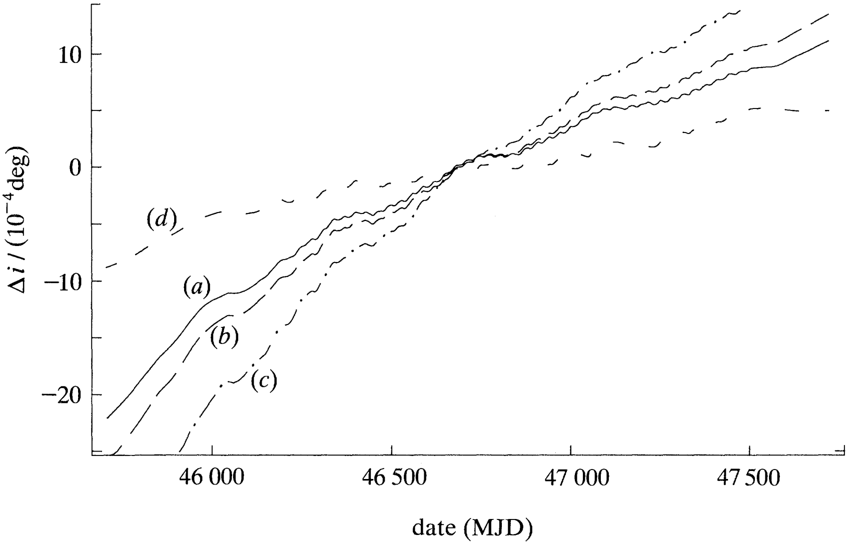

Figure 3. The perturbation in inclination from different ocean tide data and models (*a*) GEM-T2 data (Marsh *et al.* 1990), (*b*) Schwiderski ocean model from spherical decomposition of MERIT standards (Melbourne *et al.* 1983), (*c*) results from Moore (1987) Starlette data and (*d*) Williamson & Marsh (1985) Starlette data.

satellite orbit analyses. Four different sets of parameters have been compared for the total oceanic perturbation in inclination, one derived from an oceanic model and three from satellite derived results. For the oceanic model data the results of Schwiderski (1983) were used by utilizing the spherical harmonic decomposition of Merit Standards (Melbourne *et al.* 1983). The satellite data are from Williamson & Marsh (1985), Moore (1987) and the Goddard Earth Model, GEM-T2 (Marsh *et al.* 1990). The values used for the load-deformation Love numbers are $k_2' = -0.3075$, $k_4' = -0.1320$, $k_6' = -0.0892$. The resulting perturbations due to these different ocean tide models are shown in figure 3. As with the solid-Earth tides the perturbation is almost entirely due to S_2 which is resonant. The form of the perturbation is similar except that there is a difference in phase, the period of the perturbation being of the order of 10000 days, substantially longer than the time covered by the analysis. It is significant that the data from Williamson & Marsh (1985) for the S_2 tide are approximately 90° different in phase from the other models and a difference in the resulting perturbation is apparent in the figure. The problem of which data set to choose for the perturbation is compounded by uncertainties in the solar radiation pressure perturbation and is discussed in §5*e*. Best results were, however, obtained from GEM-T2 of which the tidal parameters are very well determined. The resulting amplitudes and periods of the perturbation in inclination calculated from this data set are shown in table 4, again for the beginning and end of the analysis and at exact 15th-order geopotential resonance. The amplitude is of the order of 5–10% of that due to the solid-Earth tides, and is significant. As the solid-Earth tides are well known, the possibility remains to extend the analysis away from the geopotential resonance to give an independent determination of the S_2 ocean tide amplitude and phase. Although the amplitude is still somewhat small, it is worth noting that the period is approximately 17000 days at the beginning of the analysis and in principle the parameters could be determined with good accuracy.

Table 4. *Amplitudes and periods of the principal ocean tide perturbations at the beginning and end of the analysis and at exact 15th-order geopotential resonance*

tide…	indices for ocean tide expansion	beginning of analysis (MJD 45707)				exact geopotential resonance (MJD 47353)				end of analysis (MJD 47725)			
	(rev (solar day)$^{-1}$)	$\Delta i\beta$, 2*tuv*	$\Delta i\beta$, 4*tuv*	$\Delta i\beta$, 6*tuv*	period	$\Delta i\beta$, 2*tuv*	$\Delta i\beta$, 4*tuv*	$\Delta i\beta$, 6*tuv*	period	$\Delta i\beta$, 2*tuv*	$\Delta i\beta$, 4*tuv*	$\Delta i\beta$, 6*tuv*	period
origin…	n_1 n_2 n_3 n_4 n_5 n_6	deg	deg	deg	days	deg	deg	deg	days	deg	deg	deg	days
K_1 lunar and solar	1 1 0 0 0 0 (1.002738 rev d^{-1})	3.4×10^{-5}	-1.8×10^{-5}	—	369.1	3.4×10^{-5}	-1.9×10^{-5}	—	379.2	3.4×10^{-5}	-1.9×10^{-5}	—	377.3
P_1 solar	1 1 −2 0 0 0 (0.997262 rev d^{-1})	-1.3×10^{-5}	—	—	−361.4	-1.2×10^{-5}	—	—	−352.3	-1.2×10^{-5}	—	—	−353.9
K_2 lunar and solar	2 2 0 0 0 0 (2.005476 rev d^{-1})	-1.4×10^{-5}	—	—	184.6	-1.5×10^{-5}	—	—	189.6	-1.5×10^{-5}	—	—	188.6
M_2 lunar	2 0 0 0 0 0 (1.932274 rev d^{-1})	1.2×10^{-5}	—	—	−14.8	1.2×10^{-5}	—	—	−14.7	1.2×10^{-5}	—	—	−14.7
S_2 solar	2 2 −2 0 0 0 (2.000000 rev d^{-1})	3.5×10^{-3}	-3.7×10^{-4}	9.0×10^{-5}	−17292.4	1.0×10^{-3}	-1.1×10^{-4}	2.7×10^{-5}	−4973.9	1.2×10^{-3}	-1.3×10^{-4}	3.2×10^{-5}	−5721.7

(*e*) *Solar radiation pressure*

The perturbation due to solar radiation pressure on the orbit of Meteor 28 was estimated to be of similar magnitude to that due to ocean tides and therefore had to be taken into account. The algorithm used to calculate this perturbation was that due to Aksnes (1976) which actually assumes a spherical spacecraft, but the approximation if sufficiently accurate for the purposes of this analysis. In outline the approach involves finding the shadow exit and entry positions of the satellite in its orbit and, for each revolution, integrating the perturbation between these two limits.

Lagrange's planetary equations may be written in terms of the direction cosines of a force μF, where μ is as defined in §4*a*. For the inclination and eccentricity these are

$$\left.\begin{aligned} \mathrm{d}i/\mathrm{d}t &= na^2(1-e^2)^{-\frac{1}{2}}FW(r/a)\cos u, \\ \mathrm{d}e/\mathrm{d}t &= na^2(1-e^2)^{\frac{1}{2}}F\{S(v)\sin v + T(v)[\cos v + e^{-1}(1-r/a)]\},\end{aligned}\right\} \qquad (15)$$

where v is the true anomaly and $u = v+\omega$. The force is assumed to be acting along the Sun–Earth line which in turn is assumed to be parallel to the Sun-satellite line. Any component normal to this direction is assumed negligible. The $S(v)$, $T(v)$ and W are the direction cosines of the force μF along the satellite radius vector r, perpendicular to r in the orbital plane and along the orbit normal, respectively. The force is given by

$$\mu F = s(A/m)P(a_{\odot}/r_{\odot})^2, \qquad (16)$$

where A/m is the cross-sectional area-to-mass ratio of the satellite, $P(\approx 4.65\times 10^{-6}$ N m^{-2}) is the force per unit area exerted by the Sun when its geocentric distance $r_{\odot}$ is equal to its mean distance $a_{\odot}$. The parameter s depends upon the reflection characteristics of the satellite's surface and usually has a value of between 1.0–1.5. The direction cosines $S(v)$, $T(v)$ and W are given by Kozai (1961).

The perturbations in i and e are given in terms of the eccentric anomaly, E, as (Aksnes 1976)

$$\left.\begin{aligned} \delta i &= a^2FW(1-e^2)^{-\frac{1}{2}}|[-\tfrac{3}{2}eE + (1+e^2)\sin E - \tfrac{1}{4}e\sin 2E]\cos w \\ &\quad + (1-e^2)^{\frac{1}{2}}(\cos E - \tfrac{1}{4}e\cos 2E)\sin w|_{E_1}^{E_2}, \\ \delta e &= a^2F(1-e^2)^{\frac{1}{2}}|\tfrac{1}{4}S(0)(1-e^2)^{\frac{1}{2}}\cos 2E + T(0)(\tfrac{3}{2}E - 2e\sin E + \tfrac{1}{4}\sin 2E)|_{E_1}^{E_2}.\end{aligned}\right\} \qquad (17)$$

The limits of the integrations E_1 and E_2 are the shadow exit and entrance eccentric anomalies, respectively. The integration and updating of the orbital elements is performed for each revolution. A search is required to find E_1 and E_2, the satellite being in shadow if $S(v) > 0$ and $R_{\mathrm{E}} - r\surd(1-s^2(v)) > 0$. The radius R_{E} depends on R (the Earth's equatorial radius), the Earth's flattening f_{E} and the declinations $\delta_{\odot}$ and δ of the Sun and satellite respectively, and is given by (Aksnes 1976)

$$R_{\mathrm{E}} = R[1 - f_{\mathrm{E}}(\sin\delta + S(v)\sin\delta_{\odot})^2/(1-S^2(v))]. \qquad (18)$$

Some computer time is saved by noting that the satellite will be in sunlight throughout its orbit if (but not only if) $a(1-e) > R_{\mathrm{E}}/|W|$.

The parameters s and A/m are unknown and difficult to determine with any degree of accuracy. The procedure adopted was to lump the two together into a single parameter $S_{AM}(=sA/m)$ and to find an optimal value by minimizing ϵ (the measure of fit parameter used in THROE) during the fitting process. As has been stated in §5*d* this was further complicated by the necessity of deciding upon the best ocean tide data-set to use. In principle it would have been possible to derive a value for S_{AM}

by analysing the change in semi-major axis due to solar radiation pressure. This perturbation was, however, extremely small (estimated to be of the order of 10 m for the entire period of the analysis) and so was completely 'swamped' by the change due to air drag, which in turn manifests an unknown level of uncertainty. No optimal value for S_{AM} could be found consistently for the eccentricity fitting, the results depending too much upon the number of lumped harmonics being fitted. This is regretable since the eccentricity is not significantly affected by tidal forces. However, a clear consistent minimum could be found in the fitting of the inclination, the best combination being the GEM-T2 ocean tidal data and $S_{AM} = 0.03\ \mathrm{cm}^2\ \mathrm{g}^{-1}$. This value for S_{AM} is somewhat lower than expected based on the spacecraft configuration, and cannot be regarded as rigorously determined. It is, however, consistent with the satellite being uncontrolled, that is without its solar arrays constantly directed towards the Sun. If this were the case, it is likely that there would be a small component of the solar radiation pressure force normal to the Sun–Earth line and this has not been taken into account. The perturbations in inclination and eccentricity using $S_{AM} = 0.03\ \mathrm{cm}^2\ \mathrm{g}^{-1}$ are shown in figure 4*a* and *b* respectively. It can be seen from figure 4*a* that the rate of change of the inclination declines significantly towards the end of the analysis and this corresponds to the satellite being in sunlight throughout its orbit (the resulting perturbation being short-periodic.

(*f*) *Air drag*

The perturbation due to air drag on the inclination (atmospheric rotation) was removed by THROE in the fitting process. The perturbation in the eccentricity is more significant and a more sophisticated drag model was used than the one contained in THROE for its removal. The method of King-Hele (1986) was used, working in terms of the parameter $Q = a(1-e)$. The change in Q due to drag may be written

$$\mathrm{d}Q/\mathrm{d}t = -(2a\dot{n}/3n)(1-\mathrm{d}x/\mathrm{d}a), \tag{19}$$

where $x = ae$. An expression for $\mathrm{d}x/\mathrm{d}a$ is taken from the model of Swinerd & Boulton (1982) which assumes an oblate atmosphere with a diurnal density variation. The variations in $\mathrm{d}a$ and $\mathrm{d}x$ are written, using their notation as

$$\begin{aligned}\mathrm{d}a = K\{&A_1 + eA_2 + c\cos 2w(A_5 + eA_6) + c^2(A_8 + \cos 4wA_9) \\ &+ FA[A_{14} + eA_{15} + c\cos 2wA_{18} + c^2(A_{21} + \cos 4wA_{22})] \\ &+ FB[c\sin 2wA_{27} + c^2 \sin 4wA_{30}] + O(\Gamma)\},\end{aligned} \tag{20}$$

where $O(\Gamma) = O(e^2, ce^2, c^2e, c^3, Fe^2, Fce, Fc^2e, Fc^3)$ and $\mathrm{d}x$ is of identical functional form except with A_i replaced by X_i. The A_i and X_i are functions of $z = ae/H$ and are given by Swinerd & Boulton (1982), z being evaluated at zH_p above perigee height, where H_p is the density scale height at perigee. The oblateness parameter, c, is given by

$$c = (\epsilon' a/2H)(1-e)\sin^2 i, \tag{21}$$

where $\epsilon' = 0.00335$ is the atmospheric ellipticity. The diurnal amplitude, F, is defined by (King-Hele 1987)

$$F = (f-1)/(f+1), \tag{22}$$

where f is the ratio of maximum to minimum atmospheric density.

The atmospheric model of Jacchia (1977), J77 was used to determine f at any particular height. To obtain the maximum and minimum atmospheric density, the

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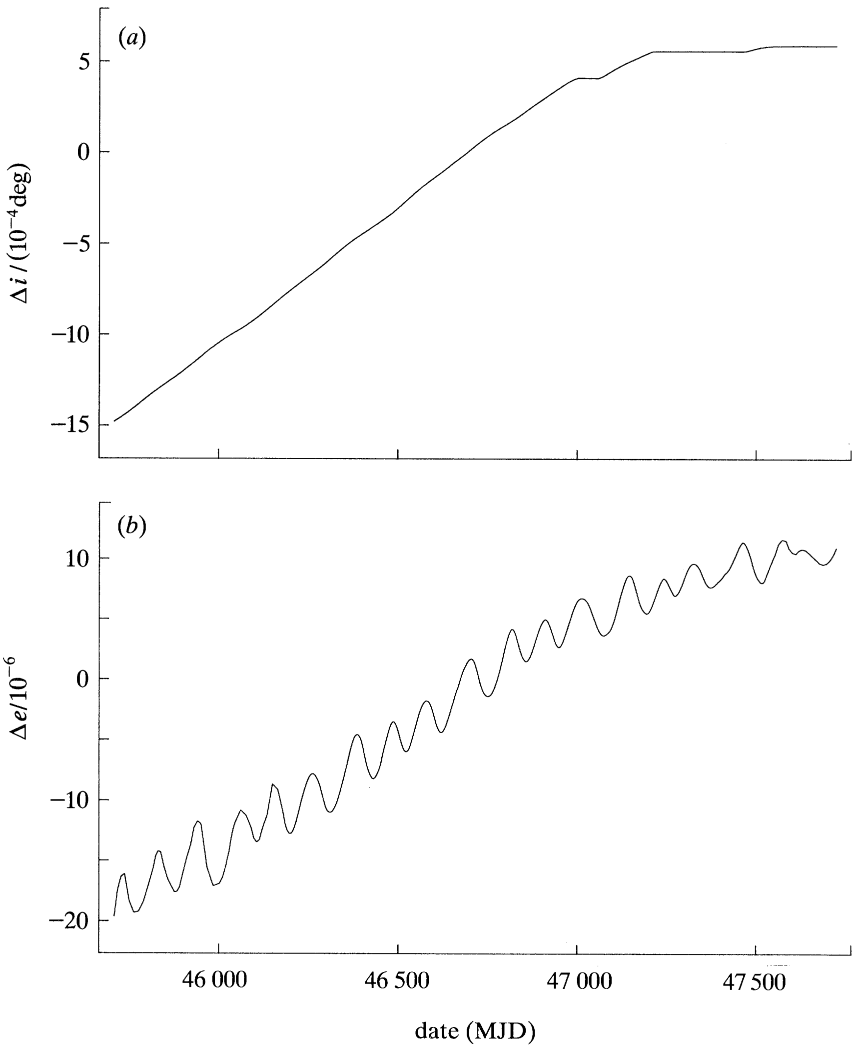

Figure 4. The perturbations in (*a*) inclination and (*b*) eccentricity due to solar radiation pressure calculated using $S_{AM} = 0.03\ \mathrm{cm^2\ g^{-1}}$.

maximum and minimum exospheric temperatures are required and these are expressed in terms of the mean temperature in the J77 model in Swinerd & Boulton (1983).

The maximum in the diurnal density is assumed to occur at 14.5 local solar time. Integration of equation (19) gives the variation in Q due to air drag and this is shown in figure 5. It can clearly be seen from the figure that the perturbation is initially small but that there is a marked increase in the drag during higher solar activity beyond MJD 47200 (February 1988). This increase caused some problems in the fitting of the eccentricity (§7).

The atmospheric perturbations were then removed from the observed Q values. The change in Q due to resonance alone may be expressed as

$$\Delta Q_{\mathrm{res}} = (1-e)\,\Delta a_{\mathrm{res}} - a\Delta e_{\mathrm{res}}. \tag{23}$$

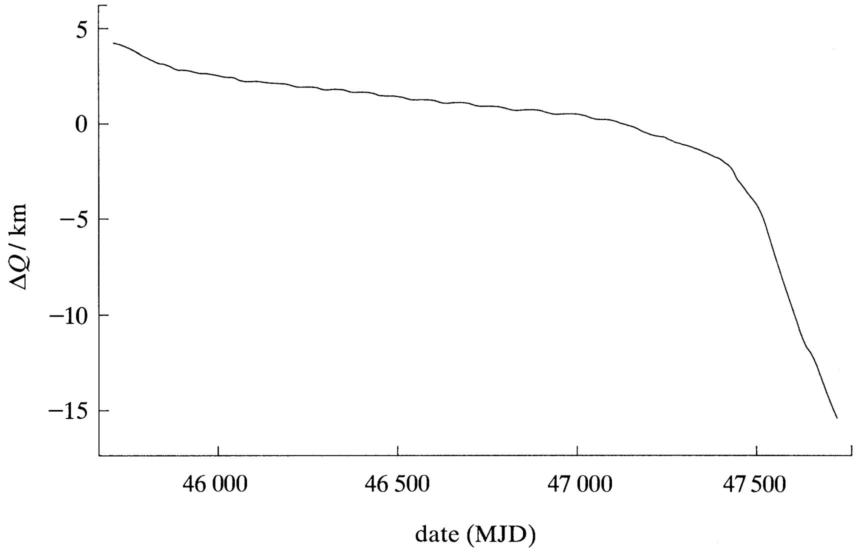

Figure 5. The perturbation in Q due to drag.

It can be shown (Allan 1973) that the change in a due to 15th-order resonance may be linked to that of i by

$$\Delta a_{\text{res}} = (-2a \sin i/(15 - \cos i))\, \Delta i_{\text{res}}. \tag{24}$$

Equation (23) may then be rewritten as

$$\Delta e_{\text{res}} = -a^{-1}[\Delta Q_{\text{res}} + (2 \sin i/(15 - \cos i))\, Q \Delta i_{\text{res}}]. \tag{25}$$

The variation in inclination due to resonance alone was used in equation (25) together with the ΔQ_{res} values to obtain Δe_{res}.

6. Analysis of inclination

(a) *The reduced form of the equation for* di/dt

For 15th-order resonance, taking the $(\gamma, q) = (1,0)$, $(2,0)$, $(1,-1)$ and $(1,1)$ terms, the equation for di/dt becomes

$$\begin{aligned} \mathrm{d}i/\mathrm{d}t = {} & (n(1-e^2)^{-\frac{1}{2}}/\sin i)\,(R/a)^{15}[\{\bar{F}_{15,15,7}\, G_{15,7,0}(\bar{C}^{0,1}_{15} \sin \Phi - \bar{S}^{0,1}_{15} \cos \Phi) \\ & + 2(R/a)^{15} \bar{F}_{30,30,14}\, G_{30,14,0}(\bar{C}^{0,2}_{30} \sin 2\Phi - \bar{S}^{0,2}_{30} \cos 2\Phi)\}\,(15 - \cos i) \\ & + 15(R/a)\,\bar{F}_{16,15,8}\, G_{16,8,1}(\bar{S}^{1,0}_{15} \sin(\Phi - \omega) + \bar{C}^{1,0}_{15} \cos(\Phi - \omega)) \\ & + (15 - 2\cos i)\,(R/a)\,\bar{F}_{16,15,7}\, G_{16,7,-1}(\bar{S}^{-1,2}_{15} \sin(\Phi + \omega) + \bar{C}^{-1,2}_{15} \cos(\Phi + \omega))]. \end{aligned} \tag{26}$$

The $\bar{F}_{lmp}$, G_{lpq}, and $Q^{q,k}_{l}$ (equation (8)) are determined computationally using mean values for the inclination, eccentricity and semi-major axis.

(b) *The fitting of the variation in inclination*

The variation in the resonance angle Φ and its rate of change $\dot{\Phi}$ are shown in figure 6. This shows that exact resonance occurred on 11 July 1988 (MJD 47353). Over the period of the analysis the rate of change of the resonance angle increased from -8.2 to 19.8 deg d^{-1}. Initially the resonant state was approached extremely slowly so the effect on the orbital elements should be very significant. However, towards the end of the analysis the increased air drag rapidly increased $\dot{\Phi}$.

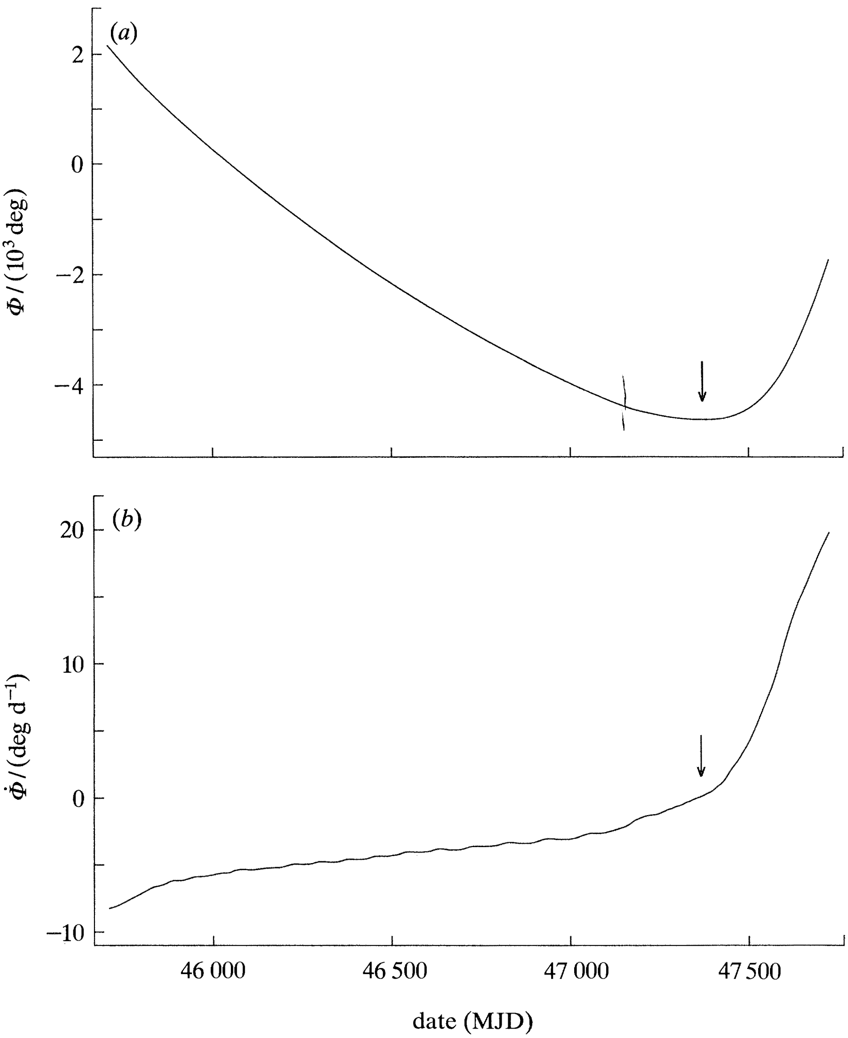

Figure 6. The variation of (a) Φ and (b) $\dot{\Phi}$ during the period of the analysis. The arrows indicate the time of exact resonance.

The perturbations were removed from the observed inclination data, the standard deviations of the orbits being relaxed to 0.000 25° to allow for neglected perturbations and the uncertainty in the solar radiation pressure parameter, S_{AM}. The M_2 values were averaged between epochs, $\bar{M}_2$ being used in preference to the values at the epochs. The inclination values were then fitted using THROE, taking the atmospheric rotation rate $\Lambda = 1.0$ rev d^{-1}. The density scale height was taken to be 66 km corresponding to a height of 600 km, which is 0.75 H above the perigee.

An initial THROE fitting with the $(\gamma, q) = (1, 0)$ and $(2, 0)$ terms gave a measure of fit $\epsilon = 3.27$. The values of the lumped harmonics obtained were

$$\left.\begin{aligned} 10^9\bar{C}^{0,\,1}_{15} &= -23.69 \pm 1.01, & 10^9\bar{S}^{0,\,1}_{15} &= -8.11 \pm 0.96, \\ 10^9\bar{C}^{0,\,2}_{30} &= 15.47 \pm 3.89, & 10^9\bar{S}^{0,\,2}_{30} &= -7.92 \pm 5.68. \end{aligned}\right\} \quad (27)$$

Further fittings were tried with additional (γ, q) terms. The addition of the $(3, 0)$ term

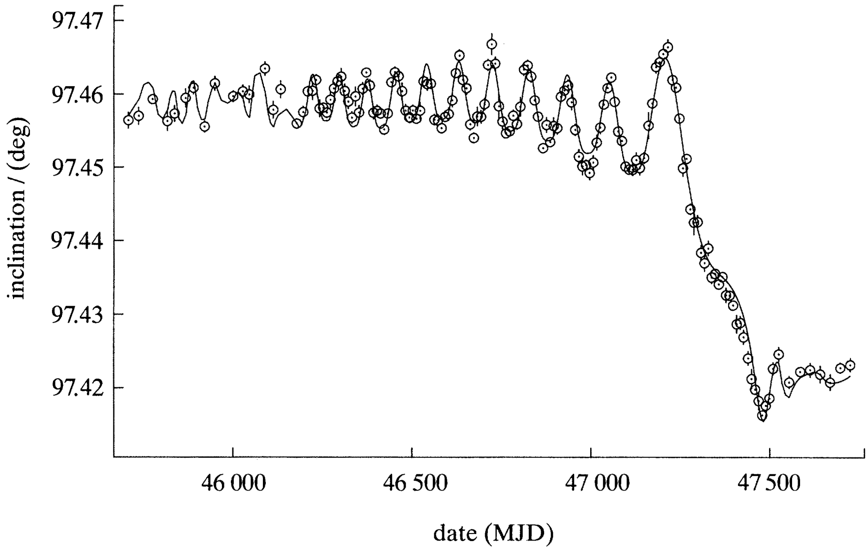

Figure 7. The variation in inclination due to 15th-order resonance. The curve gives the fitting to the data points using the THROE program with $(\gamma, q) = (1,0)$, $(2,0)$ and $(1,1)$.

did not improve the fit but the $(1,1)$ and $(1,-1)$ terms both gave improvements. However, only the addition of the $(1,1)$ term gave well-determined lumped harmonics, the values being

$$\left.\begin{array}{ll} 10^9\bar{C}^{0,1}_{15} = -25.86\pm0.73, & 10^9\bar{S}^{0,1}_{15} = -2.02\pm0.90, \\ 10^9\bar{C}^{1,0}_{15} = 6.57\pm10.97, & 10^9\bar{S}^{1,0}_{15} = -54.53\pm5.43, \\ 10^9\bar{C}^{0,2}_{30} = 32.80\pm3.07, & 10^9\bar{S}^{0,2}_{30} = -3.41\pm3.95, \end{array}\right\} \tag{28}$$

with $\epsilon = 2.26$. The fact that the $(\gamma, q) = (1,1)$ coefficients are well determined and that those of $(1,-1)$ are not is to be expected since the $\Phi-\omega$ resonance (corresponding to $(\gamma, q) = (1,1)$) was passed through at a much lower rate than that for the $\Phi+\omega$ resonance ($(\gamma, q) = (1,-1)$).

Tests were made for the sensitivity of the values of the lumped harmonics to changes in both the density scale height and the atmospheric rotation rate, Λ. No significant variations were found although the value for Λ finally taken was 0.85 (King-Hele & Walker 1988). The standard deviations of nine points were relaxed by a factor of two and of one point by a factor of four to keep all the weighted residuals less than 2ϵ. This final fit of the inclination gave $\epsilon = 1.89$ and the preferred values of the lumped harmonics were

$$\left.\begin{array}{ll} 10^9\bar{C}^{0,1}_{15} = -26.18\pm0.63, & 10^9\bar{S}^{0,1}_{15} = -2.55\pm0.78, \\ 10^9\bar{C}^{1,0}_{15} = 12.01\pm9.41, & 10^9\bar{S}^{1,0}_{15} = -51.11\pm4.75, \\ 10^9\bar{C}^{0,2}_{30} = 32.14\pm2.75, & 10^9\bar{S}^{0,2}_{30} = -0.11\pm3.46. \end{array}\right\} \tag{29}$$

The values of inclination cleared of all significant perturbations except resonance with respect to the geopotential are shown in figure 7. The theoretical curve from the THROE fitting with $(\gamma, q) = (1,0)$, $(2,0)$, $(1,1)$ that gave the values of equation (29) is also shown. As can be seen, the total change in inclination due to 15th-order resonance is approximately 0.04° and is very significant. The corresponding change in orbital position is *ca.* 5 km. The fitting is clearly very good considering all the other perturbing forces which had to be removed from the data. All the oscillations of the curve appear to be justified by the data and it is clear that the perturbational modelling for the inclination has been successful.

7. Analysis of eccentricity

(a) *The reduced form of the equation for* de/dt

For 15th-order resonance, with the $(\gamma, q) = (1, -1)$, $(1, 1)$, $(2, -1)$ and $(2, 1)$ terms, the equation for de/dt becomes

$$\begin{aligned} \mathrm{d}e/\mathrm{d}t = (n(1-e^2)^{-\frac{1}{2}}/e)\,[(R/a)^{16}\bar{F}_{16,15,7}\; G_{16,7,-1}[\bar{C}_{15}^{-1,2}\cos(\Phi+\omega)+\bar{S}_{15}^{-1,2}\sin(\Phi+\omega)] \\ -(R/a)^{16}\bar{F}_{16,15,8}\,G_{16,8,1}[\bar{C}_{15}^{1,0}\cos(\Phi-\omega)+\bar{S}_{15}^{1,0}\sin(\Phi-\omega)] \\ +(R/a)^{31}\bar{F}_{31,30,14}\,G_{31,14,-1}[\bar{C}_{30}^{-1,3}\cos(2\Phi+\omega)+\bar{S}_{30}^{-1,3}\sin(2\Phi+\omega)] \\ -(R/a)^{31}\bar{F}_{31,30,15}\,G_{31,15,1}[\bar{C}_{30}^{1,1}\cos(2\Phi-\omega)+\bar{S}_{30}^{1,1}\sin(2\Phi-\omega)]]. \end{aligned} \tag{30}$$

The coefficients $\bar{F}$, G and the Q functions are determined in a similar manner as for the inclination.

(b) *The fitting of the variation in eccentricity*

The perturbations were removed from the eccentricity data and the standard deviations relaxed to 0.00001 (corresponding to an uncertainty of around 70 m in perigee height) to allow for neglected perturbations and inadequacies in the drag and solar radiation pressure modelling processes. A further relaxation was made to 0.00002 to the last 20 data points where the air drag increased significantly. The values were then fitted using THROE with its air drag model 'turned off', the e values already having been cleared of drag effects. The density scale height was taken to be 76 km corresponding to a height of 646 km which is 1.5 H above perigee. The atmospheric rotation rate was taken to be 0.85 rev d^{-1}. An initial THROE fitting with the $(\gamma, q) = (1, -1)$ and $(1, 1)$ terms gave $\epsilon = 9.75$, the values of the lumped harmonics being

$$\left.\begin{aligned} 10^9\bar{C}_{15}^{-1,2} = 23.17 \pm 12.65, \quad 10^9\bar{S}_{15}^{-1,2} = -39.06 \pm 7.81, \\ 10^9\bar{C}_{15}^{1,0} = -36.83 \pm 2.07, \quad 10^9\bar{S}_{15}^{1,0} = -55.40 \pm 1.38. \end{aligned}\right\} \tag{31}$$

The value of ϵ is poor and the corresponding fitting showed substantial deviations from the data points. However, further fittings with addition (γ, q) terms showed $(\gamma, q) = (1, 2)$ to give a significant improvement as shown in figure 8. This reduced ϵ to 6.56, which was still not satisfactory. Deviations remained between the fitted curve and the data during the time period at the end of the analysis where the drag increases. The lumped harmonics for the corresponding $\Phi - 2\omega$ resonance were also not well determined, the resonance being passed through relatively rapidly at the beginning of the analysis. As no satisfactory fitting could be obtained to the data over the total period in a single fitting it was decided to proceed with the analysis starting at an epoch after the $\Phi - 2\omega$ resonance and to stop before the drag increase. Tests over different time periods showed the best span for the analysis to be from epochs 16 to 148 corresponding to a starting date of 23 April 1985 (MJD 46178) and finishing on 27 January 1989 (MJD 47553). The fittings were still rather poor for epochs 16 to 31 and the standard deviations of these points were relaxed further to 0.00002 to compensate. It is not clear whether the ill fitting of these data points is

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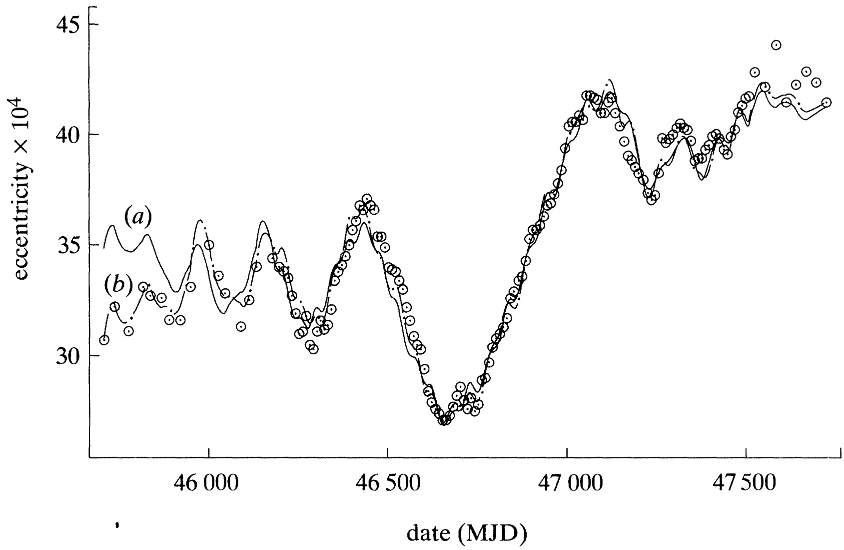

Figure 8. The variation of eccentricity due to 15th-order resonance. The curves give initial fittings to the data points using the THROE program with (*a*) $(\gamma, q) = (1, -1)$ and $(1, 1)$ $(\epsilon = 9.75)$ and (*b*) $(\gamma, q) = (1, -1)$, $(1, 1)$ and $(1, 2)$ $(\epsilon = 6.56)$.

due to perturbations which have either not been accounted for or have been inaccurately modelled. Addition of further (γ, q) terms did not improve the results in this region.

The fitting with $(\gamma, q) = (1, -1)$ and $(1, 1)$ gave $\epsilon = 6.91$, the values of the lumped harmonics being

$$\left.\begin{aligned} 10^9\bar{C}^{-1,2}_{15} &= 20.21 \pm 9.11, & 10^9\bar{S}^{-1,2}_{15} &= -39.42 \pm 7.21, \\ 10^9\bar{C}^{1,0}_{15} &= -42.08 \pm 1.57, & 10^9\bar{S}^{1,0}_{15} &= -57.27 \pm 0.99. \end{aligned}\right\} \qquad (32)$$

Three of the four values have not changed significantly from equations (31) although they are slightly better determined. Further fittings showed $(\gamma, q) = (2, 1)$ to give an improvement with $\epsilon = 6.29$. The values obtained for the lumped harmonics were

$$\left.\begin{aligned} 10^9\bar{C}^{-1,2}_{15} &= 1.37 \pm 9.24, & 10^9\bar{S}^{-1,2}_{15} &= -36.24 \pm 6.78, \\ 10^9\bar{C}^{1,0}_{15} &= -44.42 \pm 1.50, & 10^9\bar{S}^{1,0}_{15} &= -57.75 \pm 1.10, \\ 10^9\bar{C}^{1,1}_{30} &= 19.76 \pm 7.64, & 10^9\bar{S}^{1,1}_{30} &= -8.89 \pm 5.58. \end{aligned}\right\} \qquad (33)$$

It can be seen that the additional term has significantly affected the value obtained for $\bar{C}^{-1,2}_{15}$. However, the $\bar{C}^{1,1}_{30}$ and $\bar{S}^{1,1}_{30}$ values are reasonably determined and should, therefore, be included. A final run with the standard deviations increased by a factor of two on 14 of the points to keep all weighted residuals less than 2ϵ gave $\epsilon = 4.39$. This gave the preferred values of the lumped harmonics as

$$\left.\begin{aligned} 10^9\bar{C}^{-1,2}_{15} &= 0.56 \pm 6.74, & 10^9\bar{S}^{-1,2}_{15} &= -41.70 \pm 4.90, \\ 10^9\bar{C}^{1,0}_{15} &= -45.83 \pm 1.10, & 10^9\bar{S}^{1,0}_{15} &= -57.96 \pm 0.81, \\ 10^9\bar{C}^{1,1}_{30} &= 20.89 \pm 5.43, & 10^9\bar{S}^{1,1}_{30} &= -9.84 \pm 3.97. \end{aligned}\right\} \qquad (34)$$

The values were not changed significantly by variations in the density scale height. The eccentricity data cleared of all the significant perturbations except for resonance

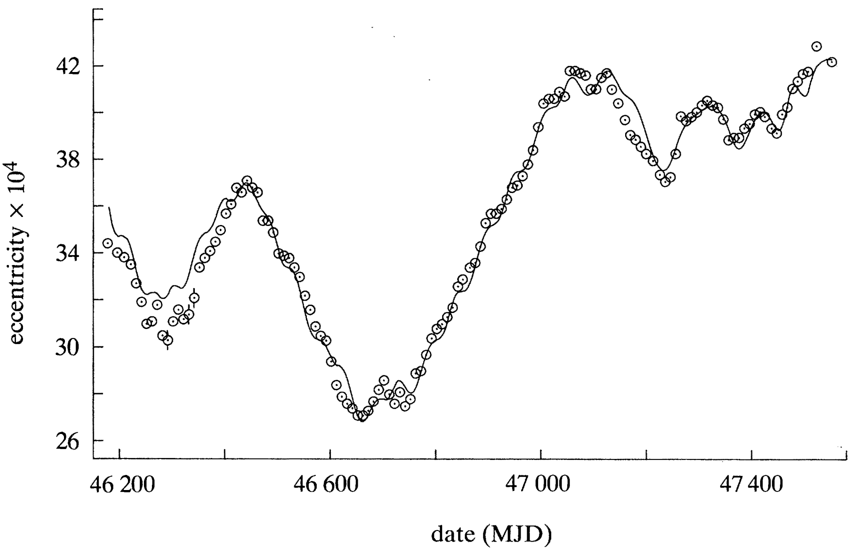

Figure 9. The preferred fitting of the variation of eccentricity due to 15th-order resonance. The curve gives the fitting to the data points using the THROE program with $(\gamma, q) = (1, -1)$, $(1, 1)$ and $(2, 1)$.

with the geopotential are shown in figure 9. The THROE fitting which gave the values of equations (34) is also shown. The fitting is not as satisfactory as that for the inclination (figure 7), a value of $\epsilon = 4.39$ being rather high. It is suspected that not all the perturbing forces have been accounted for or that some of those modelled may not be determined with sufficient accuracy. It is noteworthy that the removal of the air drag from the eccentricity data is difficult to achieve with an accuracy of better than 5–10%.

8. Simultaneous fitting of inclination and eccentricity

The inclination and eccentricity may be fitted simultaneously by using the RAE program SIMRES. The program combines the outputs of a number of THROE runs to produce a single set of lumped harmonic coefficients. The results of THROE runs with $(\gamma, q) = (1, 0)$, $(2, 0)$, $(1, -1)$, $(1, 1)$ and $(2, 1)$ for both the inclination and eccentricity were used. These separate fittings gave $\epsilon = 1.71$ and $\epsilon = 4.17$ for the inclination and eccentricity respectively. The weighting of the eccentricity in the corresponding SIMRES run was therefore downgraded by a factor equal to the ratio of the ϵ resulting from the two THROE fittings, namely 2.44. The values of the lumped harmonics were

$$\left.\begin{aligned}
10^9\bar{C}^{0,\,1}_{15} &= -26.10 \pm 0.67, & 10^9\bar{S}^{0,\,1}_{15} &= -4.02 \pm 0.66,\\
10^9\bar{C}^{-1,\,2}_{15} &= 4.51 \pm 7.63, & 10^9\bar{S}^{-1,\,2}_{15} &= -44.45 \pm 5.52,\\
10^9\bar{C}^{1,\,0}_{15} &= -44.29 \pm 1.26, & 10^9\bar{S}^{1,\,0}_{15} &= -57.64 \pm 0.89,\\
10^9\bar{C}^{0,\,2}_{30} &= 25.30 \pm 2.66, & 10^9\bar{S}^{0,\,2}_{30} &= -1.57 \pm 3.77,\\
10^9\bar{C}^{1,\,1}_{30} &= 17.02 \pm 6.25, & 10^9\bar{S}^{1,\,1}_{30} &= -8.56 \pm 4.46,
\end{aligned}\right\} \tag{35}$$

with $\epsilon = 2.09$.

The values for $\bar{C}^{1,0}_{15}$ from the individual runs differ significantly, the $\bar{C}^{1,0}_{15}$ from the inclination being not so well determined. However, the $(\gamma, q) = (1, 1)$ term is necessary in both to produce adequate fittings. For these reasons the values from the SIMRES run (equations (35)) are taken as the final preferred values.

9. Equations for the individual harmonic coefficients

The lumped harmonics may be expressed as linear combinations of the individual tesseral harmonic coefficients as demonstrated by equations (7). The mean values for the inclination, eccentricity and semi-major axis for 1977-57A are 97.4518°, 0.003 547 and 6945.25 km respectively, giving the equations as

$$\left.\begin{aligned}
\bar{C}^{0,1}_{15} = {} & \bar{C}_{15,15} - 0.1038\bar{C}_{17,15} - 0.3824\bar{C}_{19,15} - 0.4075\bar{C}_{21,15} - 0.3364\bar{C}_{23,15} \\
& - 0.2372\bar{C}_{25,15} - 0.1417\bar{C}_{27,15} - 0.0634\bar{C}_{29,15} - 0.0061\bar{C}_{31,15} \\
& + 0.0310\bar{C}_{33,15} + 0.0512\bar{C}_{35,15} + \ldots, \\
\bar{C}^{1,0}_{15} = {} & \bar{C}_{16,15} + 0.9424\bar{C}_{18,15} + 0.7593\bar{C}_{20,15} + 0.5411\bar{C}_{22,15} + 0.3287\bar{C}_{24,15} \\
& + 0.1437\bar{C}_{26,15} - 0.0033\bar{C}_{28,15} - 0.1089\bar{C}_{30,15} - 0.1752\bar{C}_{32,15} \\
& - 0.2071\bar{C}_{34,15} - 0.2115\bar{C}_{36,15} + \ldots, \\
\bar{C}^{-1,2}_{15} = {} & \bar{C}_{16,15} + 0.3813\bar{C}_{18,15} - 0.0852\bar{C}_{20,15} - 0.3405\bar{C}_{22,15} - 0.4272\bar{C}_{24,15} \\
& - 0.4020\bar{C}_{26,15} - 0.3151\bar{C}_{28,15} - 0.2045\bar{C}_{30,15} - 0.0959\bar{C}_{32,15} \\
& - 0.0044\bar{C}_{34,15} + 0.0628\bar{C}_{36,15} + \ldots, \\
\bar{C}^{0,2}_{30} = {} & \bar{C}_{30,30} - 0.7907\bar{C}_{32,30} - 0.7935\bar{C}_{34,30} - 0.4496\bar{C}_{36,30} - 0.1215\bar{C}_{38,30} \\
& + 0.0956\bar{C}_{40,30} + 0.2005\bar{C}_{42,30} + 0.2223\bar{C}_{44,30} \\
& + 0.1938\bar{C}_{46,30} + 0.1420\bar{C}_{48,30} + \ldots, \\
\bar{C}^{1,1}_{30,} = {} & \bar{C}_{31,30} + 0.4889\bar{C}_{33,30} + 0.0749\bar{C}_{35,30} - 0.1744\bar{C}_{37,30} \\
& - 0.2847C_{39,30} - 0.2982\bar{C}_{41,30} - 0.2541\bar{C}_{43,30} - 0.1834\bar{C}_{45,30} + \ldots.
\end{aligned}\right\} \quad (36)$$

The equations for the lumped $\bar{S}^{q,k}_{15}$ and $\bar{S}^{q,k}_{30}$ harmonics are identical with those for $\bar{C}$ except with C being replaced by S. Termination of the above equations has been taken at 36th-degree unless the expected contribution of the higher terms is above 5% of that due to the first, in which case the additional terms have been evaluated.

10. Approximate accuracy in geoid height

The accuracy in geoid height, σ_g, corresponding to a standard deviation σ in one of the lumped harmonics is approximately $R\sigma/Q$ (King-Hele 1986) where

$$Q = \{\Sigma(Q^{q,k}_l l_0^2/l^2)^2\}^{\frac{1}{2}}.$$

The values of σ_g (in centimetres) for each lumped harmonic are

$$\begin{aligned}
&\bar{C}^{0,1}_{15} = 0.4, \quad \bar{C}^{-1,2}_{15} = 4.4, \quad \bar{C}^{1,0}_{15} = 0.6, \quad \bar{C}^{0,2}_{30} = 1.6, \quad \bar{C}^{1,1}_{30} = 2.8, \\
&\bar{S}^{0,1}_{15} = 0.4, \quad \bar{S}^{-1,2}_{15} = 3.2, \quad \bar{S}^{1,0}_{15} = 0.4, \quad \bar{S}^{0,2}_{30} = 2.2, \quad \bar{S}^{1,1}_{30} = 2.0.
\end{aligned}$$

Thus the accuracies in geoid height for $\bar{C}^{0,1}_{15}$, $\bar{S}^{0,1}_{15}$, $\bar{C}^{1,0}_{15}$ and $\bar{S}^{1,0}_{15}$ are better than 1 cm.

Table 5. *Comparison of the values of the lumped 15th- and 30th-order harmonics from 1977-57A and those for comprehensive geoid models*

lumped harmonic	1977-57A	GEM 10B	GRIM3-L1	GEMT1	PGS-4393 (GEM T3)	KHW 89	KHW 90
$10^9\bar{C}^{0,1}_{15}$	-26.1 ± 0.7	-23.8 ± 4.9	-31.9 ± 5.8	-23.3 ± 3.9	-26.1 ± 1.2	-27.2 ± 0.8	—
$10^9\bar{S}^{0,1}_{15}$	-4.0 ± 0.7	-3.1 ± 4.9	2.4 ± 5.8	-8.2 ± 3.9	-1.2 ± 1.2	-7.7 ± 1.0	—
$10^9\bar{C}^{-1,2}_{15}$	4.5 ± 7.6	-2.5 ± 5.3	4.3 ± 6.0	-3.8 ± 5.7	0.6 ± 1.4	7.0 ± 1.9	—
$10^9\bar{S}^{-1,2}_{15}$	-44.5 ± 5.5	-18.1 ± 5.3	-20.3 ± 6.0	-21.1 ± 5.7	-21.4 ± 1.4	-12.3 ± 1.4	—
$10^9\bar{C}^{1,0}_{15}$	-44.3 ± 1.3	-62.1 ± 7.0	-59.4 ± 7.5	-49.4 ± 6.6	-55.2 ± 1.7	-64.1 ± 2.6	—
$10^9\bar{S}^{1,0}_{15}$	-57.6 ± 0.9	-43.3 ± 7.0	-57.9 ± 7.5	-52.5 ± 6.6	-56.4 ± 1.7	-47.2 ± 1.9	—
$10^9\bar{C}^{0,2}_{30}$	25.3 ± 2.7	6.4 ± 6.3	26.5 ± 4.6	-2.5 ± 8.8	21.7 ± 2.9	15.2 ± 2.7	22.4 ± 2.5
$10^9\bar{S}^{0,2}_{30}$	-1.6 ± 3.8	10.8 ± 6.3	4.5 ± 4.6	-0.9 ± 8.8	3.9 ± 2.9	5.7 ± 3.1	5.6 ± 2.3
$10^9\bar{C}^{1,1}_{30}$	17.0 ± 6.3	-0.5 ± 4.5	-2.7 ± 3.3	-0.9 ± 6.3	-0.5 ± 2.5	—	—
$10^9\bar{S}^{1,1}_{30}$	-8.6 ± 4.5	-13.4 ± 4.5	-15.5 ± 3.3	1.6 ± 6.3	-11.0 ± 2.5	—	—

11. Comparison with comprehensive geopotential models

A comparison is shown in table 5 between the lumped harmonics obtained from this analysis and those for comprehensive geoid models. The models are GEM-10B (Lerch *et al.* 1981), GRIM3-L1 (Reigber *et al.* 1985), GEM-T1 (Marsh *et al.* 1988) and PGS4393 (Marsh *et al.* 1989), a preliminary version of GEM-T3. Also included are the values computed from the King-Hele & Walker 15th- and 30th-order solution (King-Hele & Walker 1989) and 30th-order solution (King-Hele & Walker 1990), which we refer to as KHW89 and KHW90 respectively. These models, with the exception of PGS4393 and the KWH90 solutions all extend to degree and order 36. The 30th-order solution of KHW90 extends to degree 40 and PGS4393 to degree and order 50.

In general the lumped harmonics derived from this analysis are well determined and compare favourably with the model values. It is noteworthy that the value of $\bar{C}^{0,1}_{15}$ is precisely that of PGS4393 and better determined, and that $\bar{S}^{0,1}_{15}$ is about half-way between the two best previous values (which are inconsistent with each other). The values of $\bar{C}^{1,0}_{15}$ and $\bar{S}^{1,0}_{15}$ derived from 1977-57A have lower standard deviations than in any of the models: the S value agrees with PGS4393, but the C value is numerically smaller than in any of the models. The values of $\bar{C}^{0,2}_{30}$ and $\bar{S}^{0,2}_{30}$ are consistent with, and of similar accuracy to, those of PGS4393 and KHW90. Some of the less-well-determined values, such as the $\bar{S}^{-1,2}_{15}$ and $\bar{C}^{1,1}_{30}$ coefficients do differ significantly and consistently from the model values. All of the lumped harmonics determined in this analysis are sufficiently accurate to contribute to future solutions for the 15th- and 30th-order tesseral harmonic coefficients.

12. Conclusions

The orbit of satellite Meteor 28 (1977-57A) has been determined at 154 epochs between 8 January 1984 and 18 July 1989 during which it experienced 15th-order resonance with the geopotential. Almost 10000 observations were used in the orbit determination process. The orbit was Sun-synchronous and hence was significantly perturbed by many solar-induced perturbations. All the known significant perturbations have been calculated and subtracted from the inclination and eccentricity data. The removal of the solar-resonant perturbations, which were much enhanced due to the nature of the orbit, has been a major part of this analysis. The changes in the inclination and eccentricity due to resonance with the 15th- and 30th-

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order harmonics in the geopotential have been analysed to obtain six lumped harmonics of order 15 and four of order 30. The preferred values are given by equations (35). The equations relating these values to the individual tesseral harmonic coefficients are given by equations (36). The results compare favourably with values obtained from comprehensive geoid models and correspond to accuracies in geoid height of between 0.4 and 4.4 cm. The values of the lumped harmonics together with the corresponding equations will be used in a future determination of the individual 15th- and 30th-order tesseral harmonic coefficients.

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